Discrimination of correlated and entangling quantum channels with selective process tomography

Eugene Dumitrescu

QMATH 2016
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Phys. Rev. A: Out this week (2016) — arXiv:1608.01416
Collaborators: Travis Humble

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Outline

1. Direct process tomography with stabilizer codes
2. Concatenation for error detection
3. Applications to channel discrimination
4. Conclusions and Future Directions

Quantum Codes

Quantum Codes

Quantum Code:
Partitioning of Hilbert space

$$\mathcal{S} = \langle \hat{g}_1, \hat{g}_2, ..., \hat{g}_{n-k} \rangle$$

$$\mathcal{C} = \{ |\psi\rangle \ s.t. \ g_i|\psi\rangle = |\psi\rangle \ \forall i \}$$

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Quantum Error Correction:
Procedure to “combat” decoherence

$$\{\mathbb{E}_i\} \in \mathcal{P}_n = \{I, X, Y, Z\}^{\otimes n}$$

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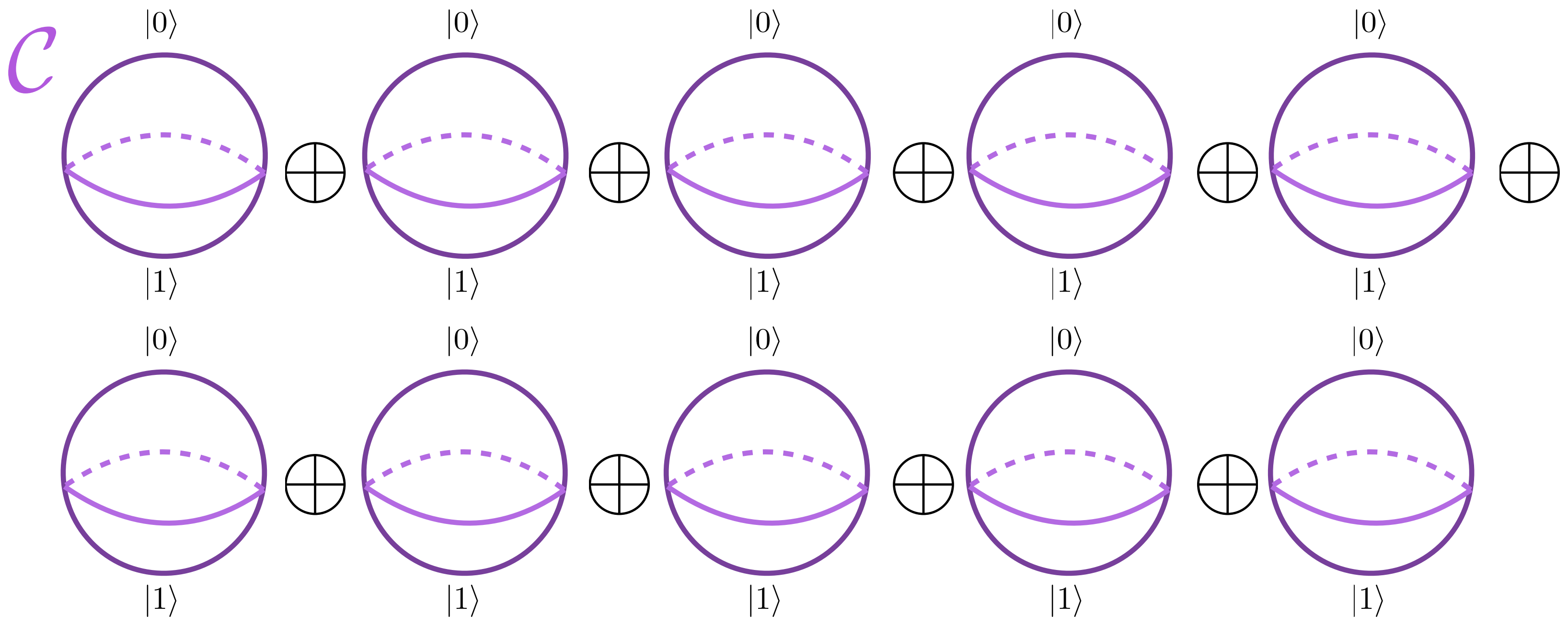

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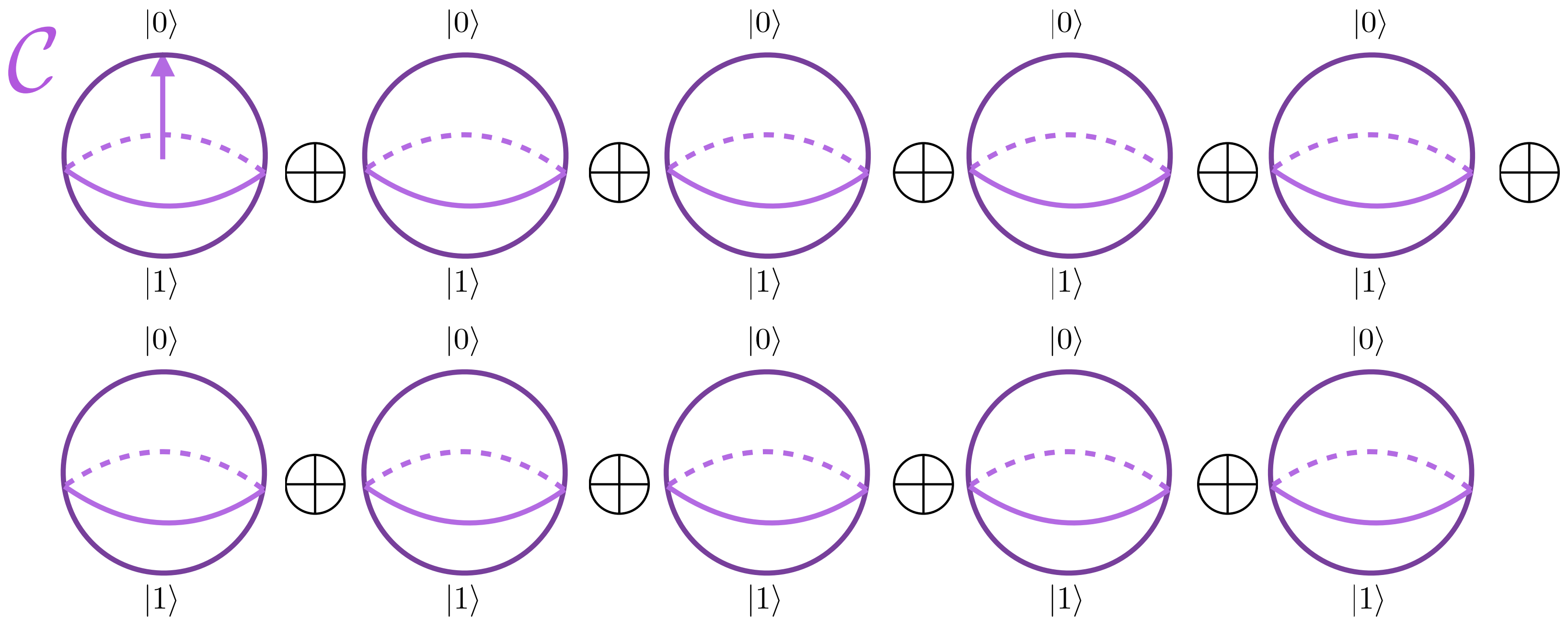

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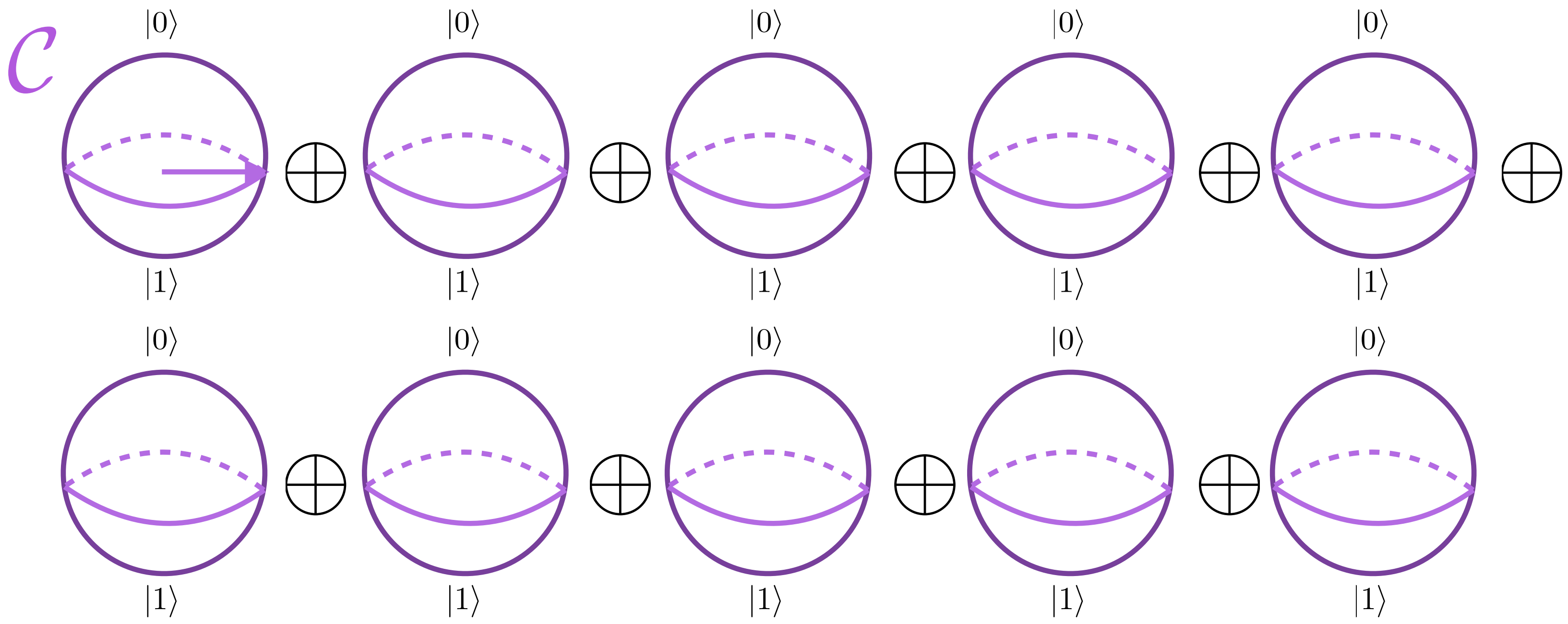

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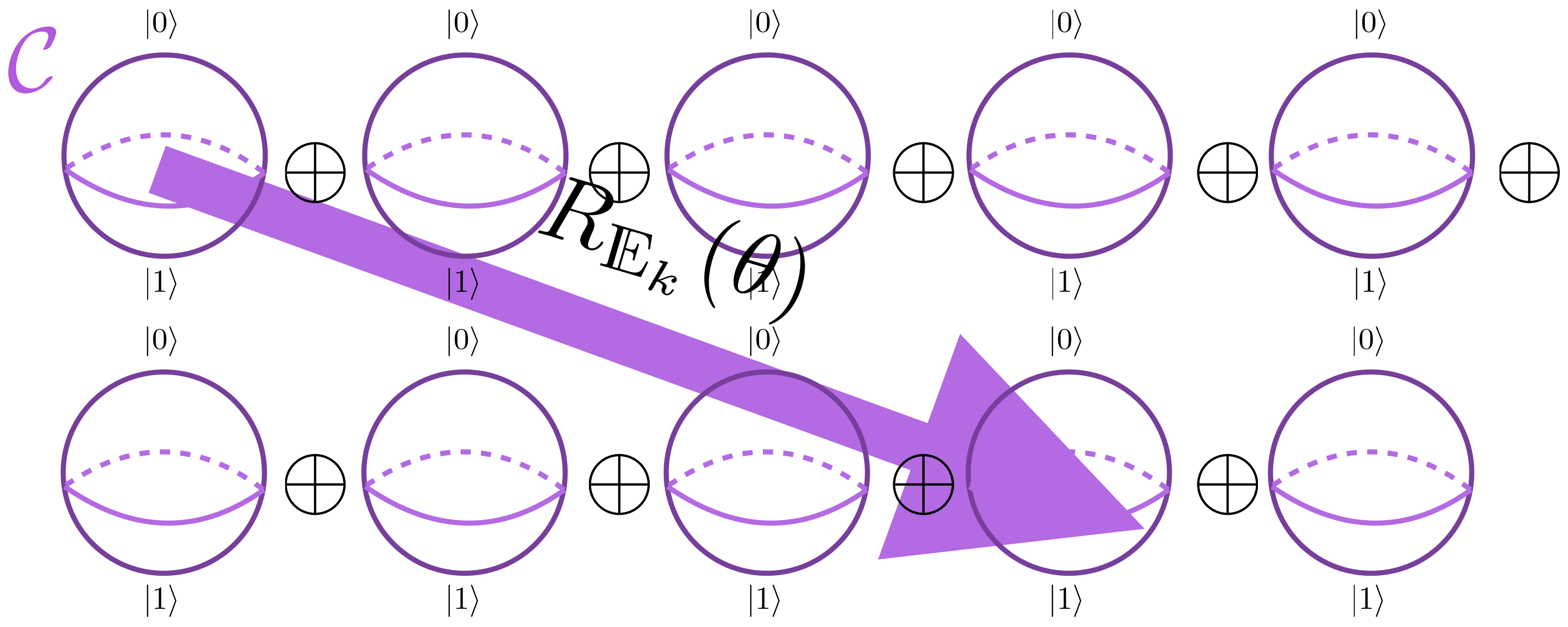

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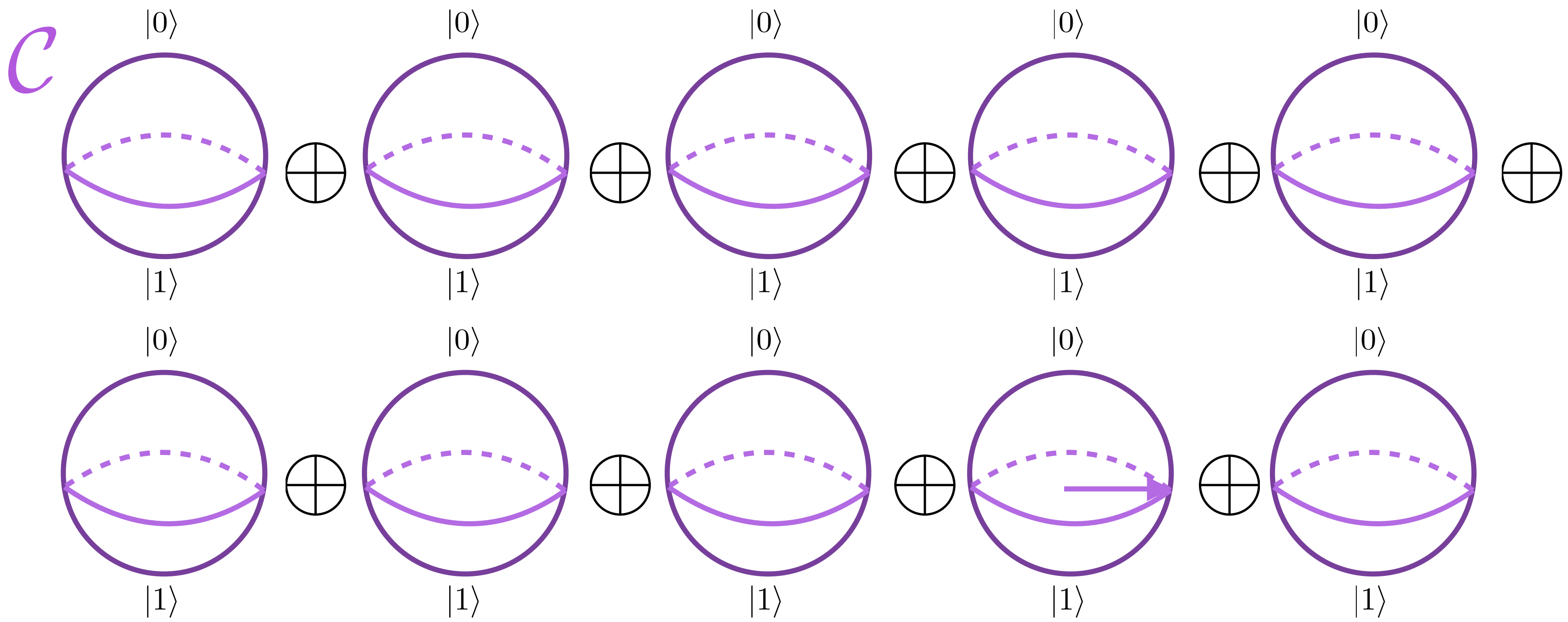

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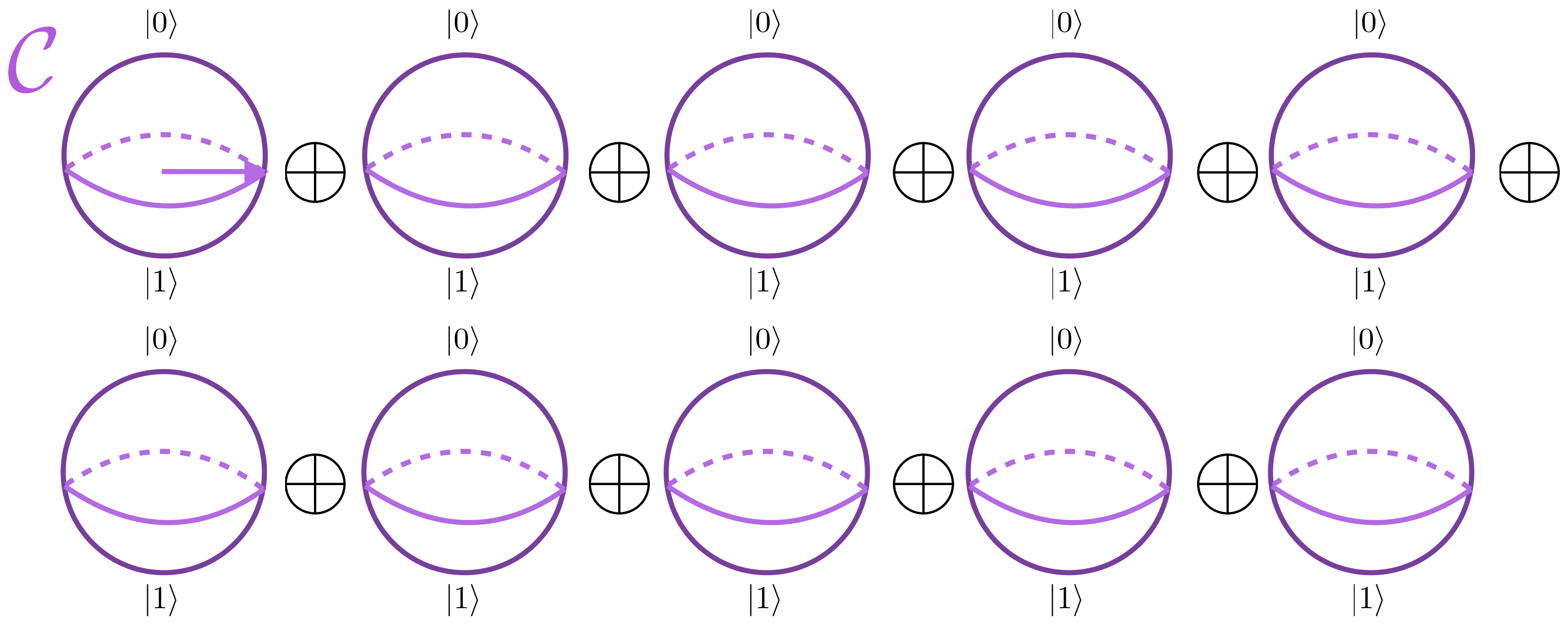

Quantum Channel

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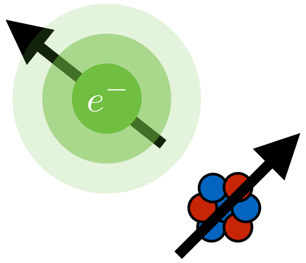

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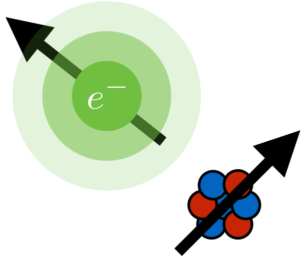

$$i\hbar\partial_t\psi(t) = \hat{H}\psi(t)$$

$$\hat{H} = \hat{H}_{sys}$$

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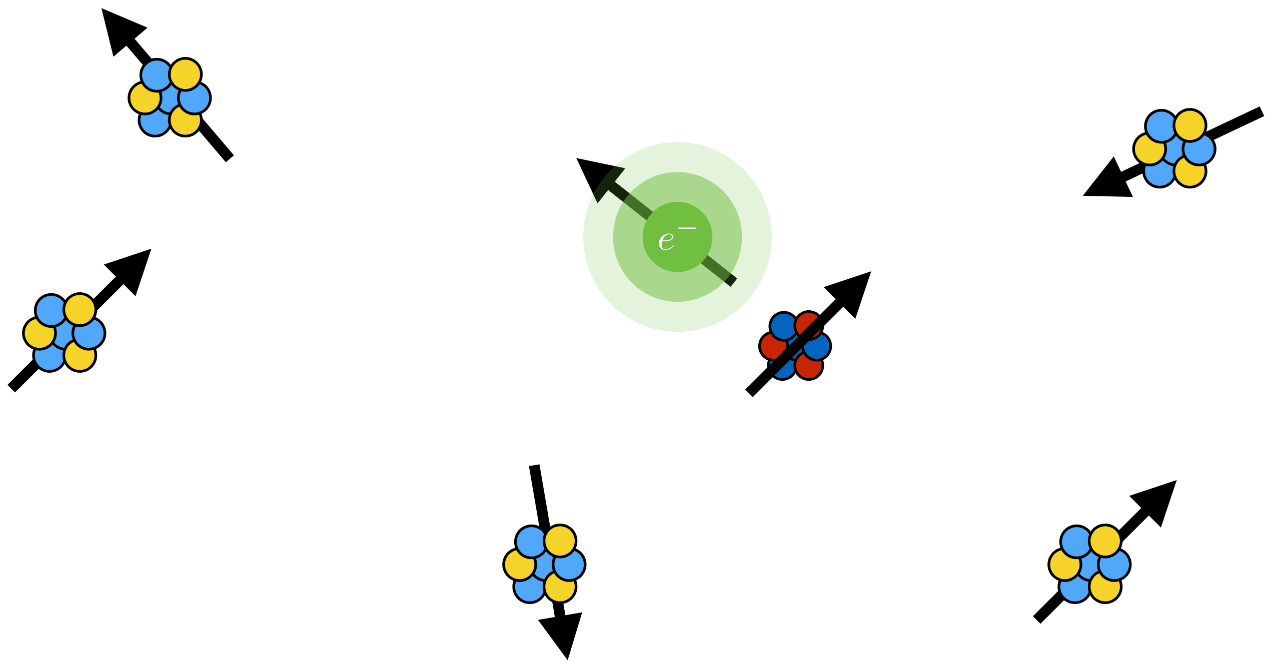

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$$+\hat{H}_{env} + \hat{H}_{sys-env}$$

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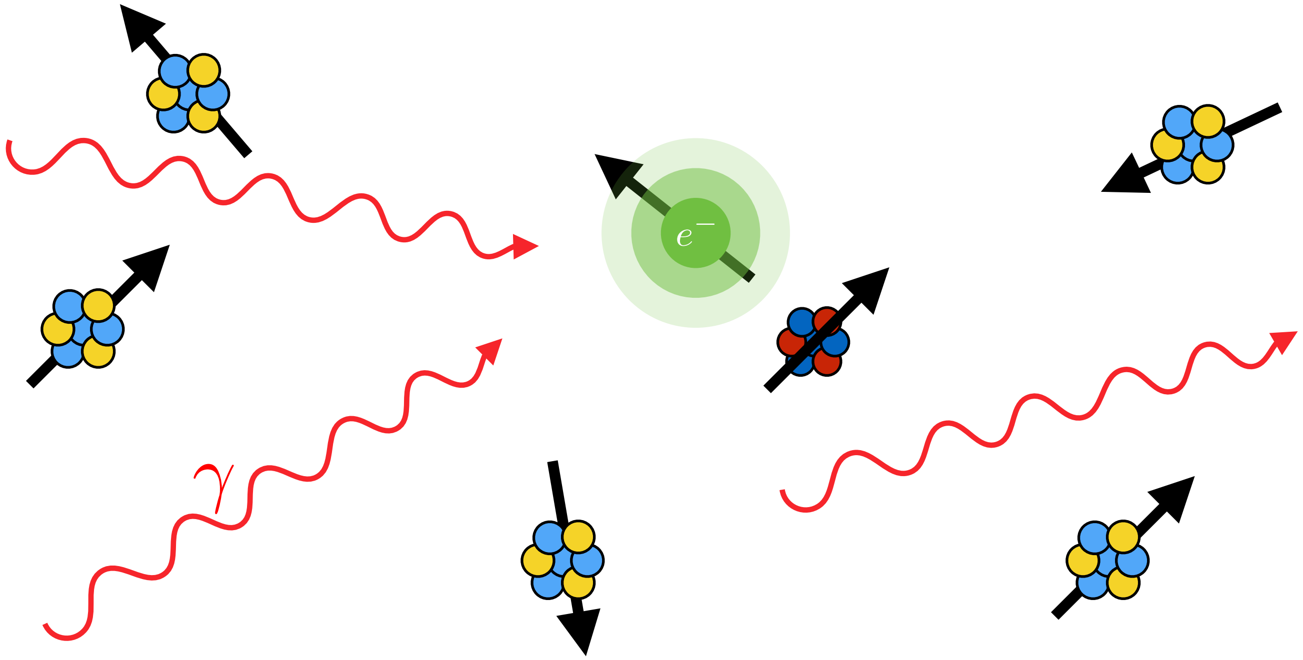

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$$+\hat{H}_{cont-env}$$

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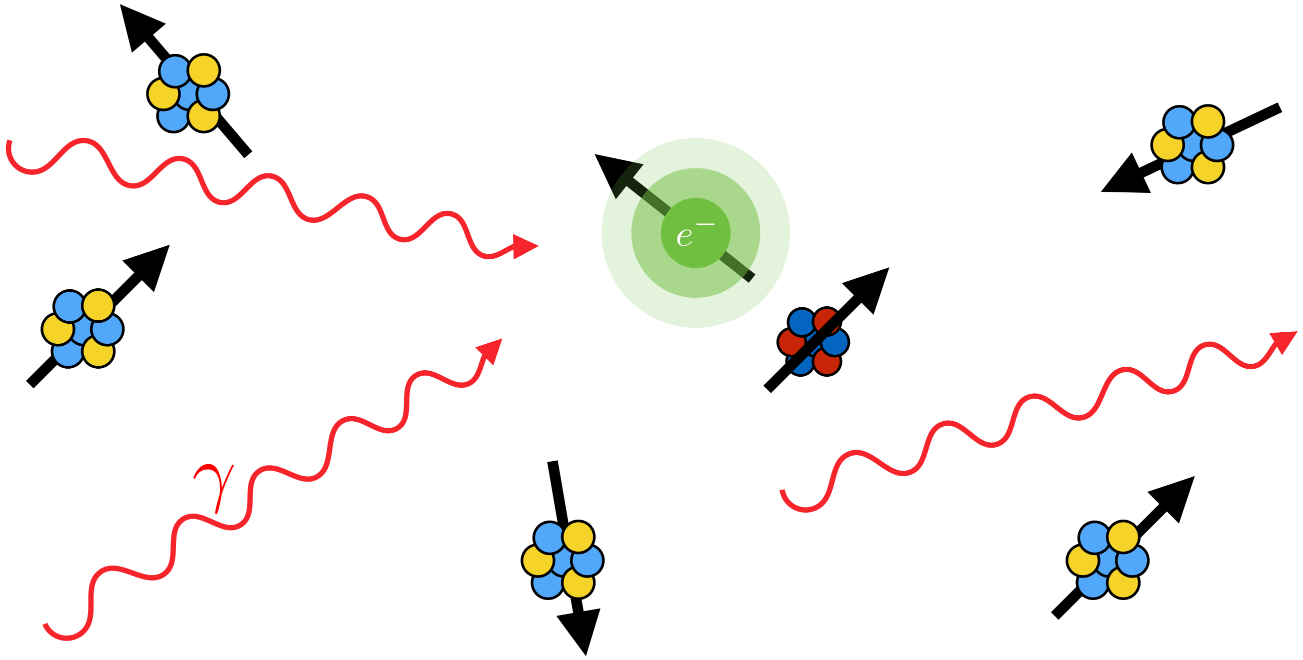

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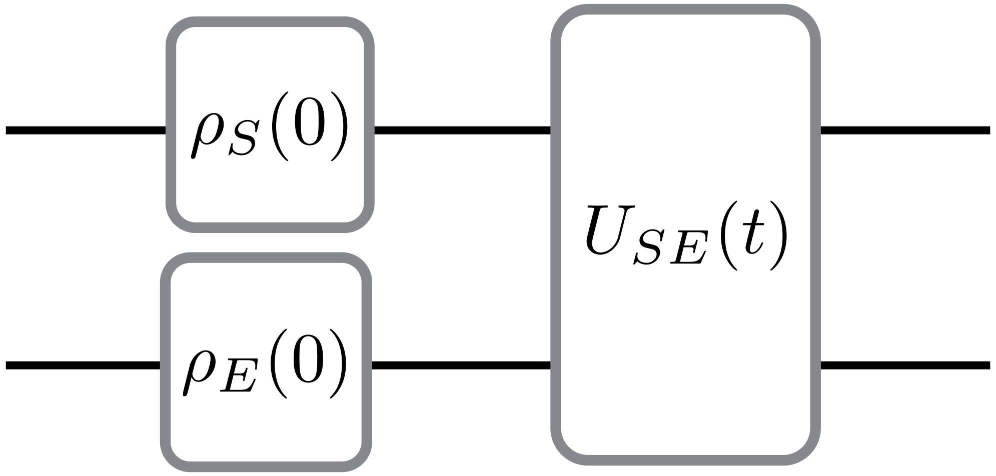
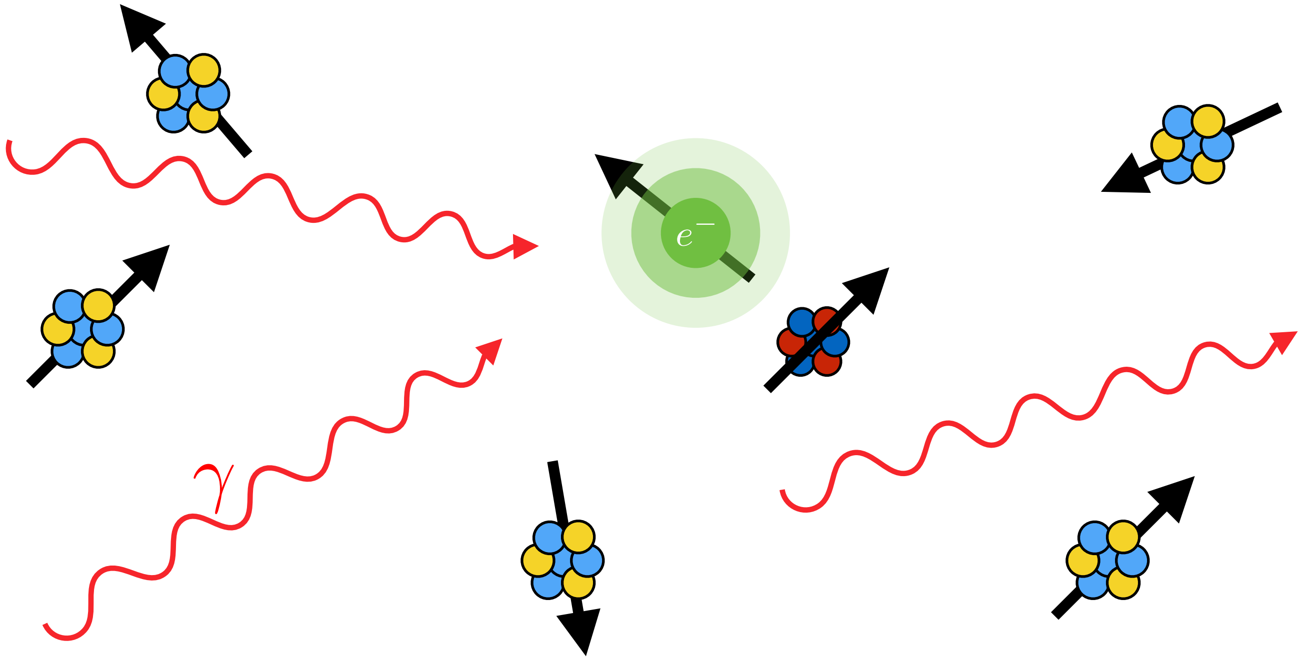


$\rho_S(0)$

$\rho_E(0)$

$U_{SE}(t)$

Quantum Channel



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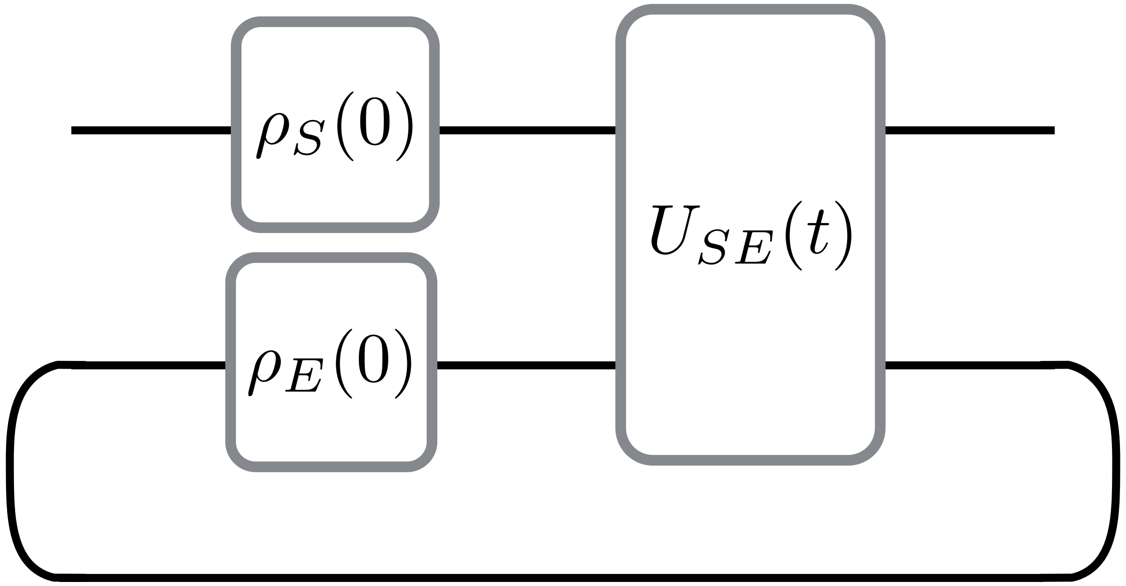

$$\rho_S(t) = \mathrm{Tr}_E\{\rho_{SE}(t)\}$$

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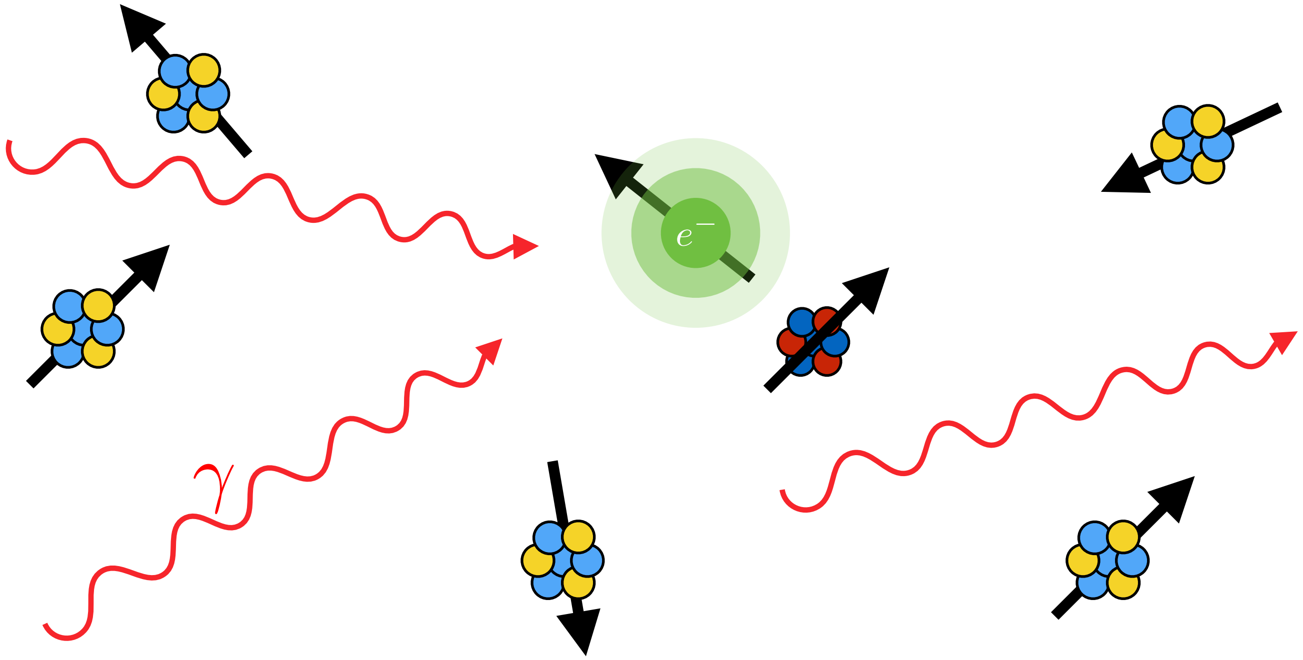

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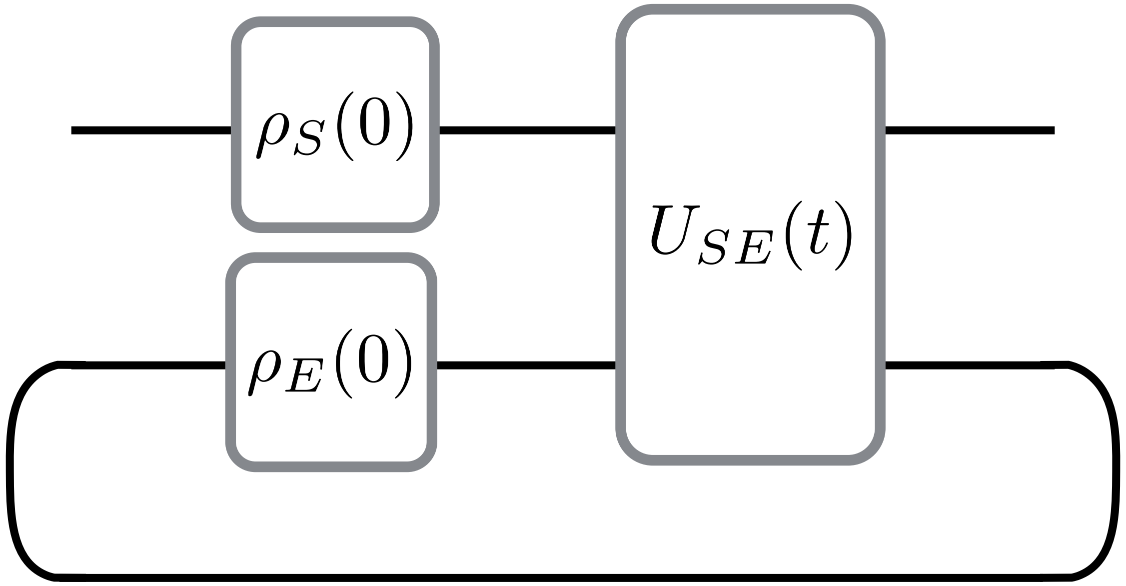

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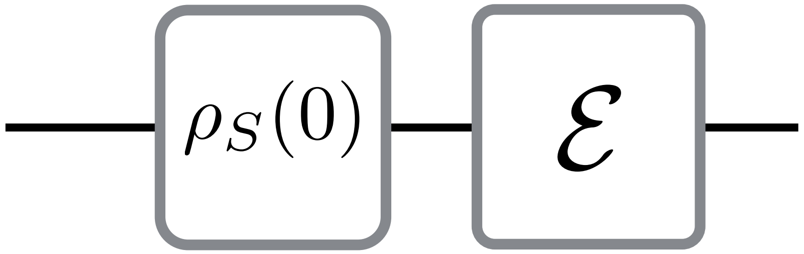
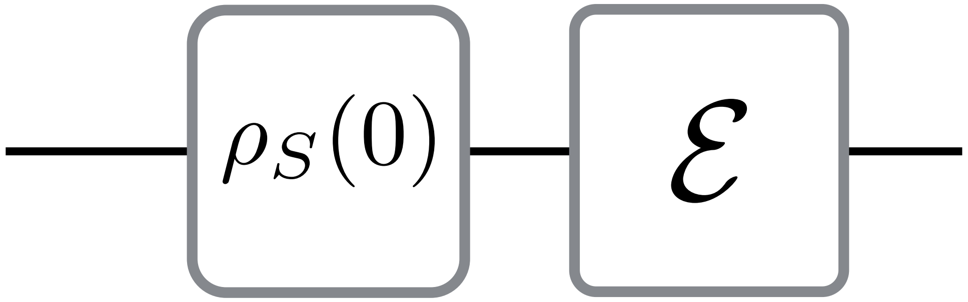


$$\mathcal{E} : \rho_i \mapsto \rho_f$$

$$\mathcal{E}(\rho_s) = \sum_i K_i \rho_s K_i^\dagger$$

Channel Structure

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger$$



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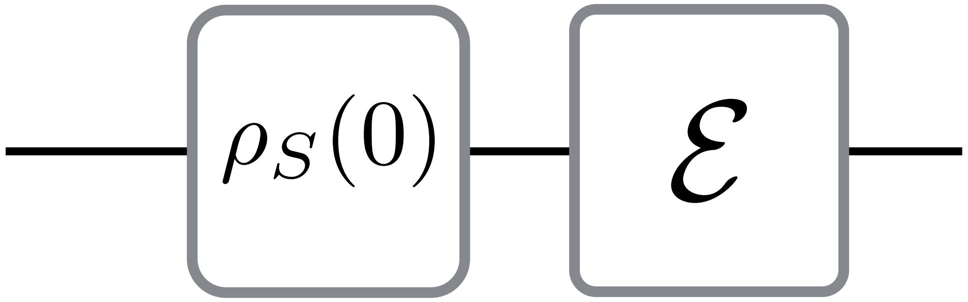

ρ : d×d positive semi-definite linear operator

K_i : d×d linear operators

$$K_i = \sum_{m=1}^{d^2} e_{im} F_m$$

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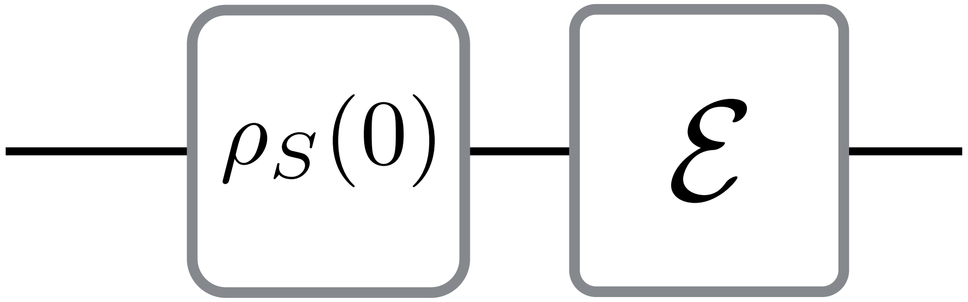

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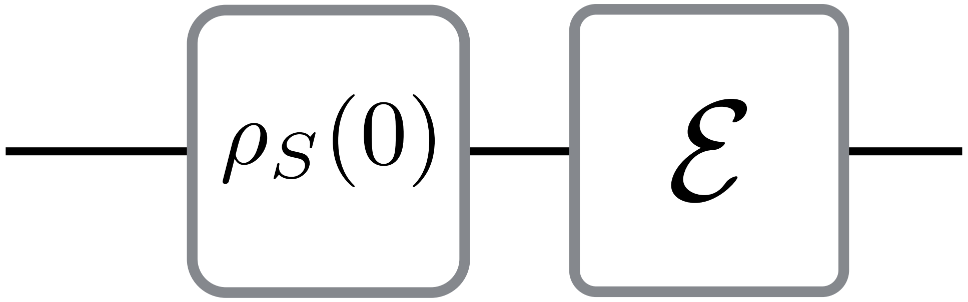

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Complete description

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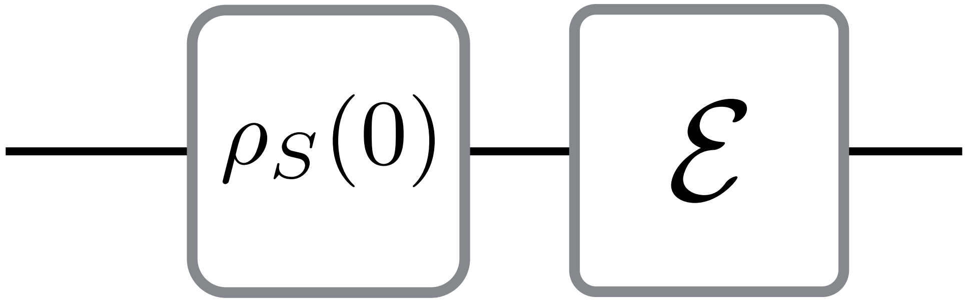

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Complete description

Informational Completeness

$$\chi_{mn} = \sum_i e_{im}^* e_{in}$$

$$\sum_i K_i^\dagger K_i = 1 = \sum_{mn} \chi_{mn} F_m^\dagger F_n$$

d^4 elements - d^2 constraints

$\mathcal{O}(d^4) = \mathcal{O}(16^n)$ measurements

Channel Tomography

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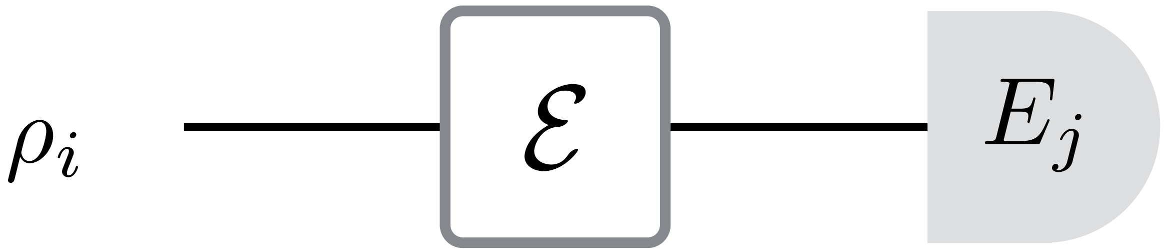

$\rho_i, E_j \in \{\Pi_0, \Pi_1, \Pi_+, \Pi_{+i}\}$

SQPT

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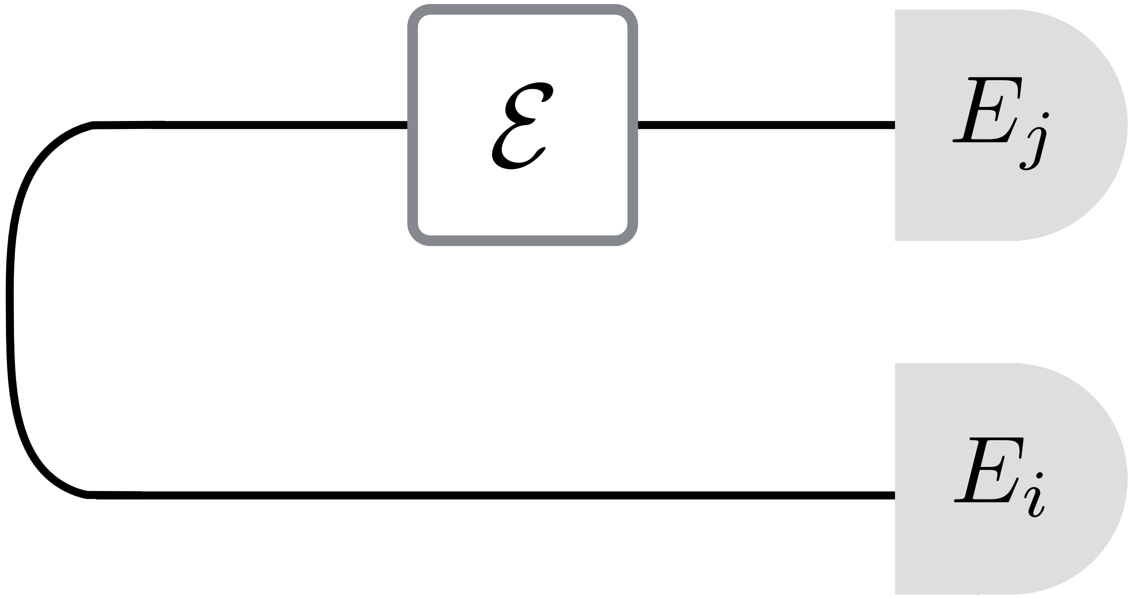

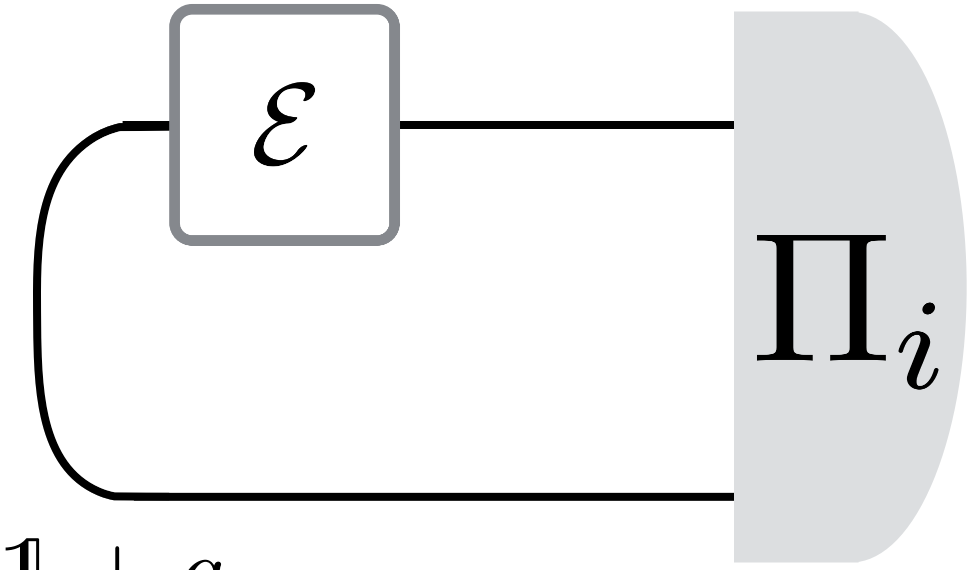

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AAPT

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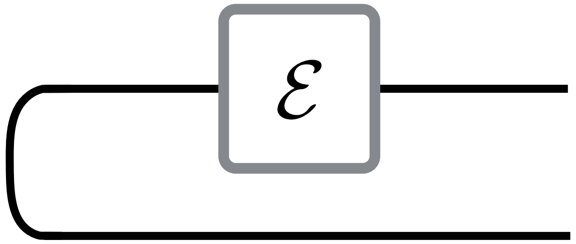
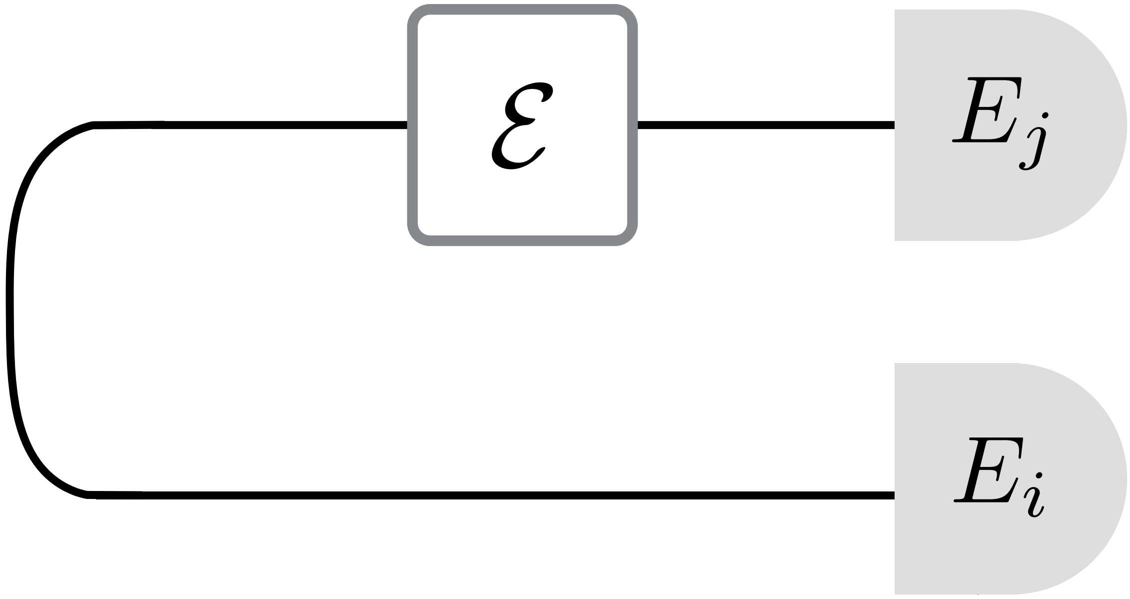


channel $\sim$ state

$\mathcal{E} \sim \mathcal{E} \otimes \mathbb{I}(\rho_{max})$

Channel Tomography



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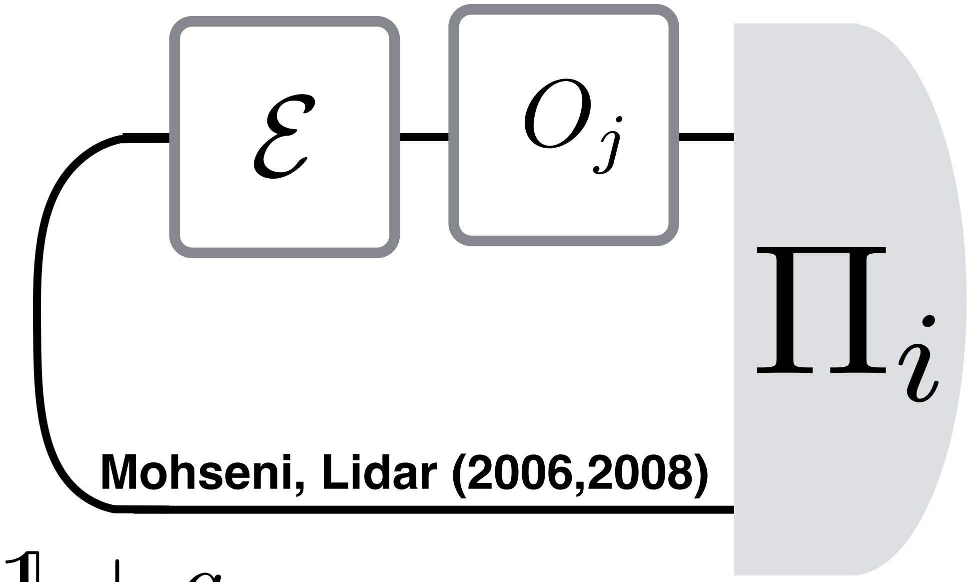

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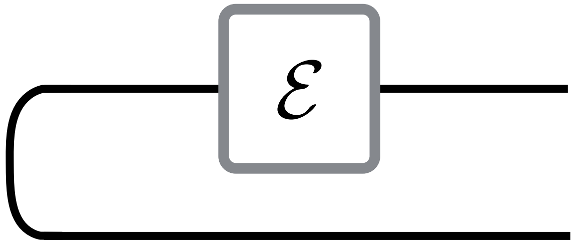

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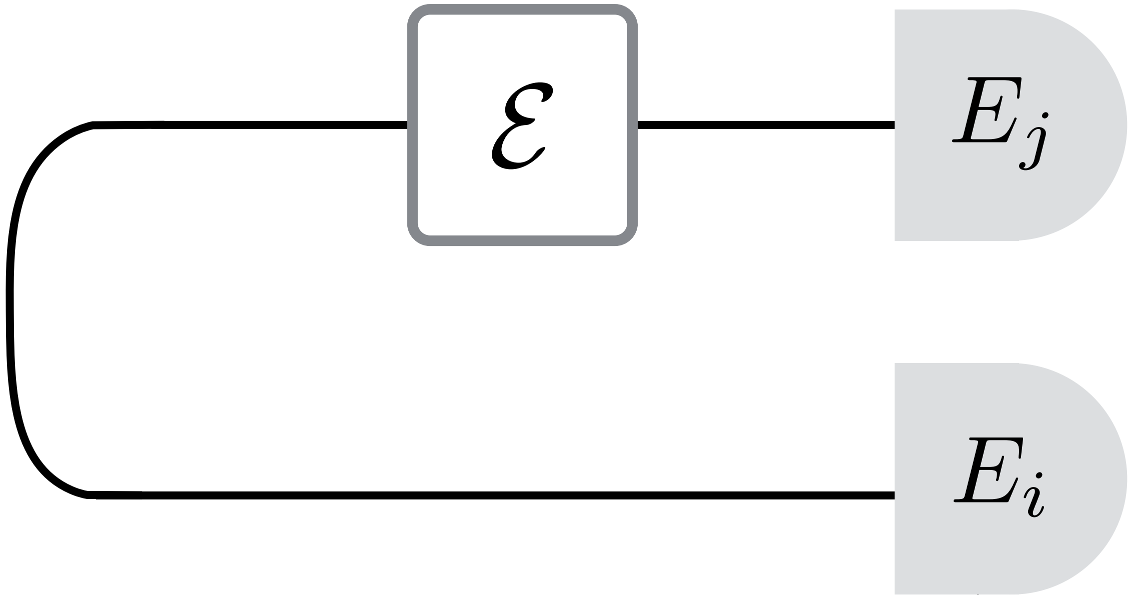

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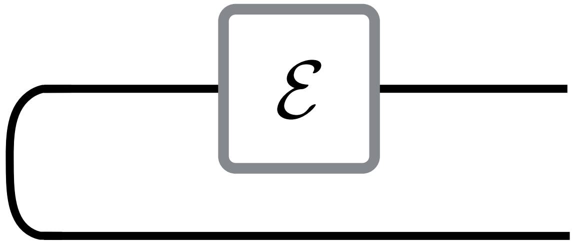

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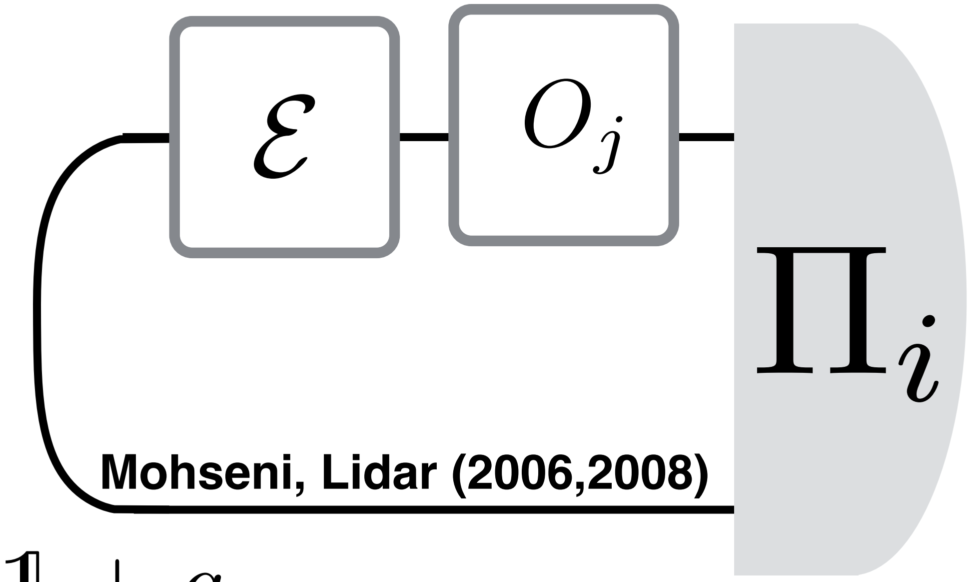

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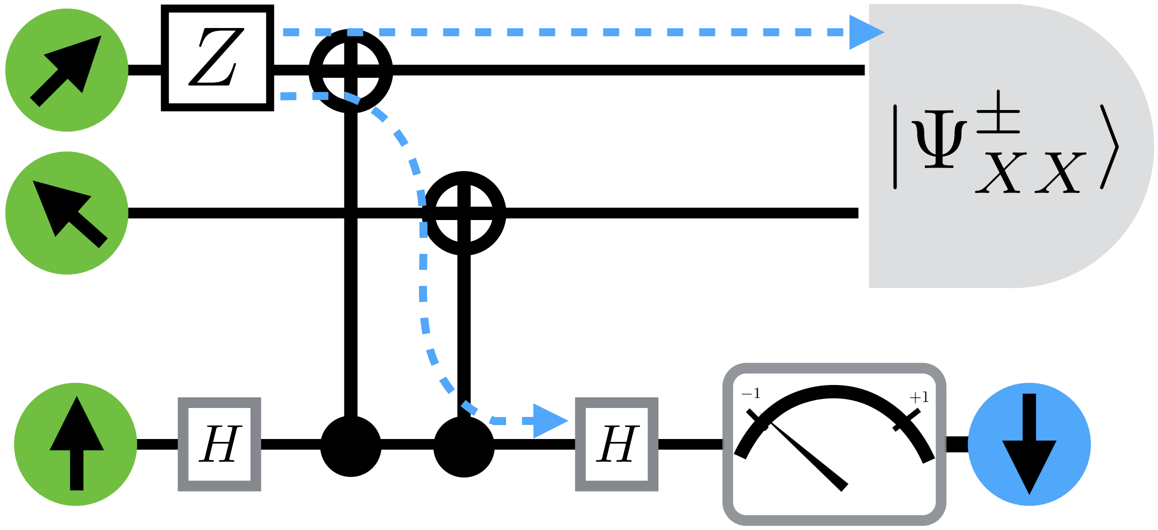

Entanglement Assisted Codes

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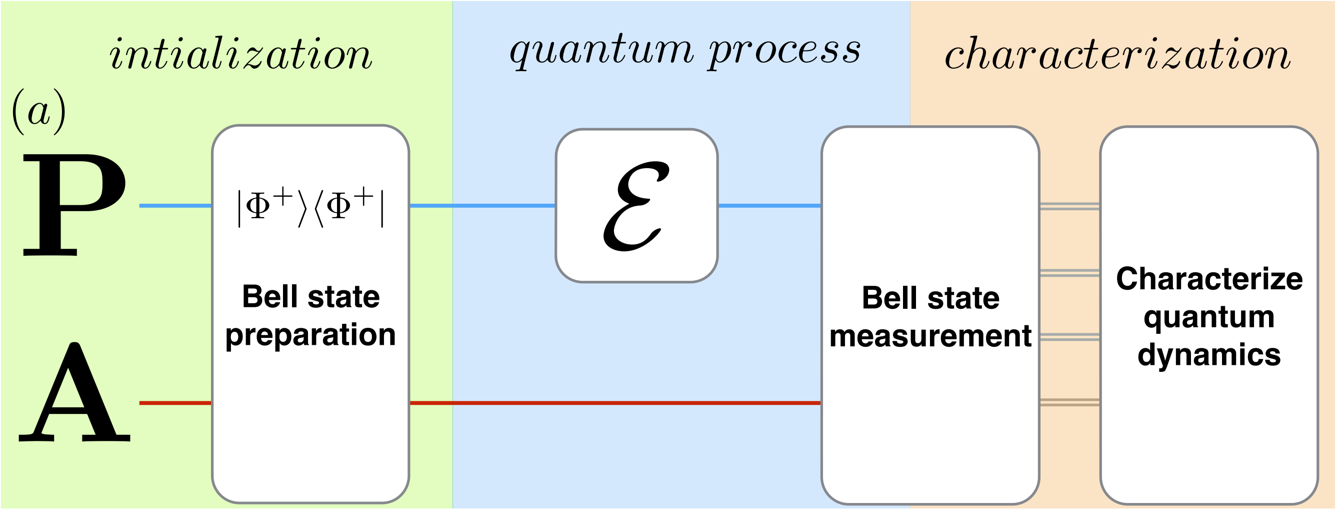
Generators: $\mathcal{S} = \langle XX, ZZ \rangle$



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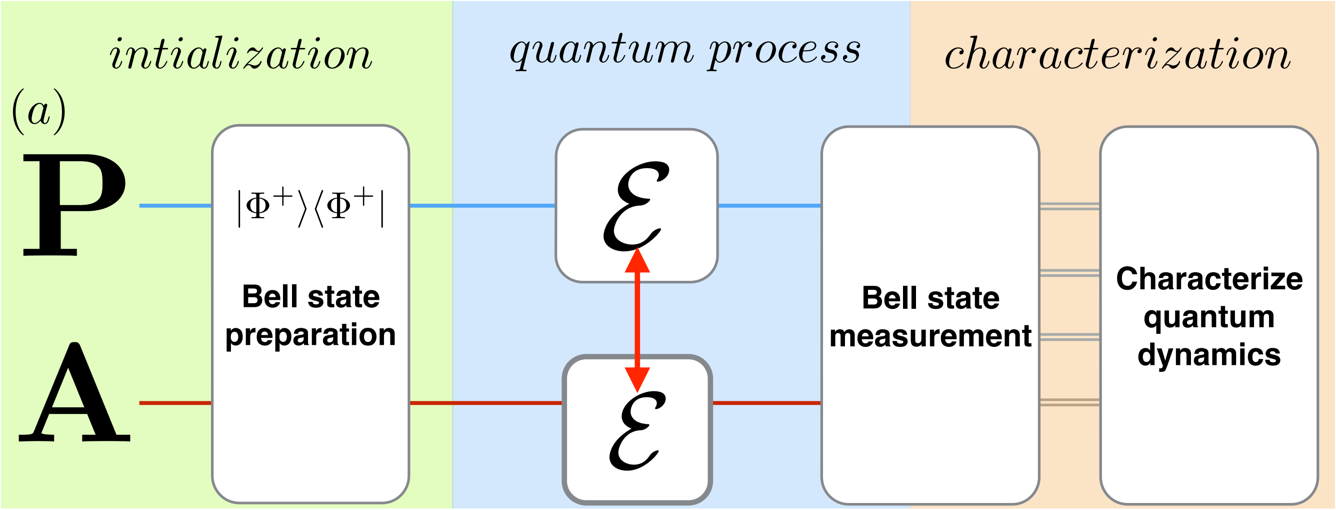
$Z_1, Z_2 : |\Phi^+\rangle \mapsto |\Phi^-\rangle$



Entanglement Assisted Codes

Codestate: $|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Generators: $\mathcal{S} = \langle XX, ZZ \rangle$

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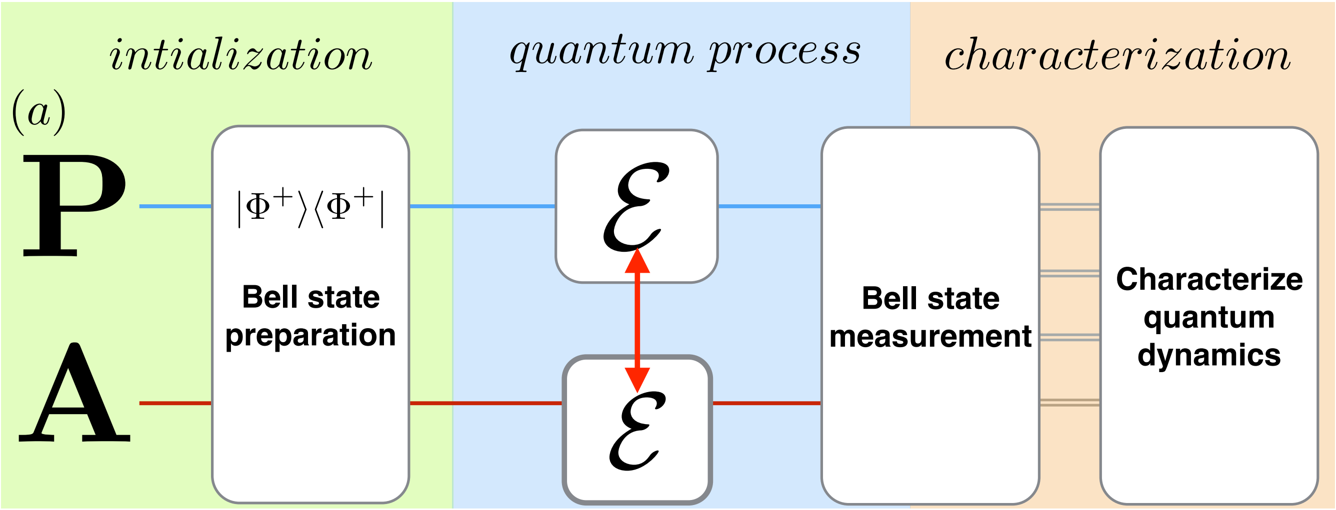

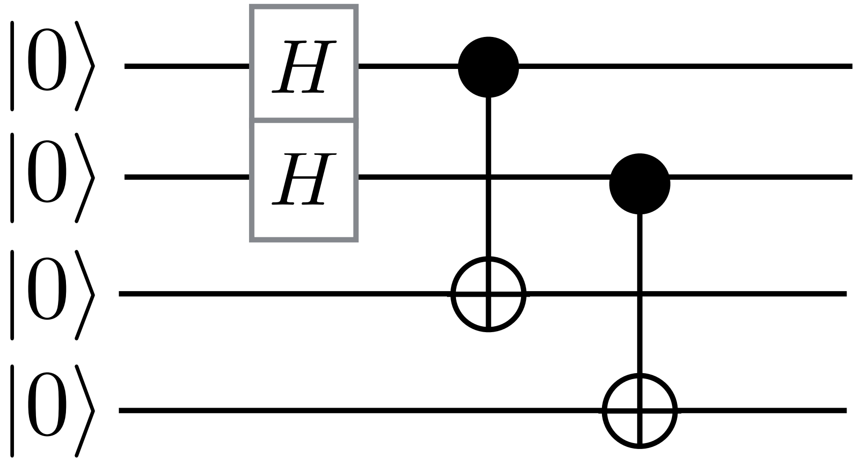

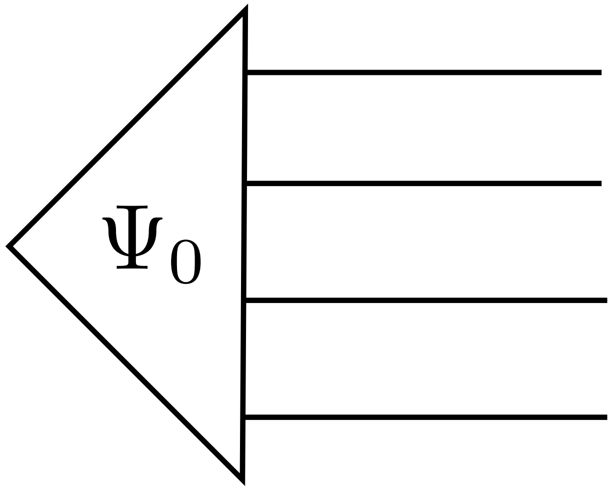

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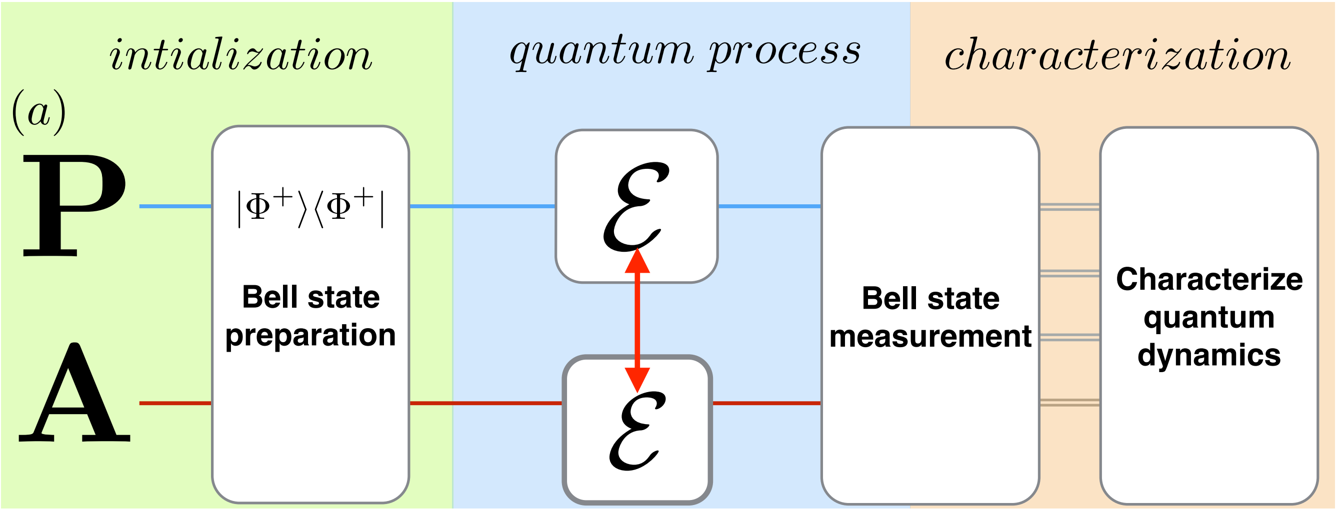

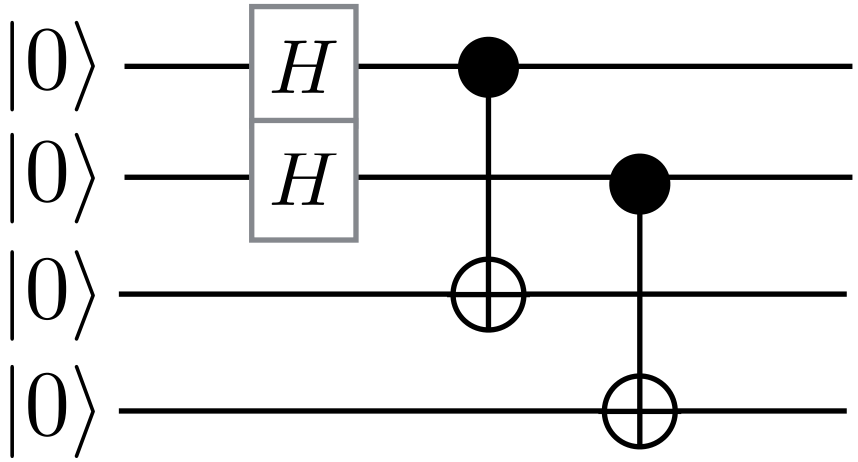

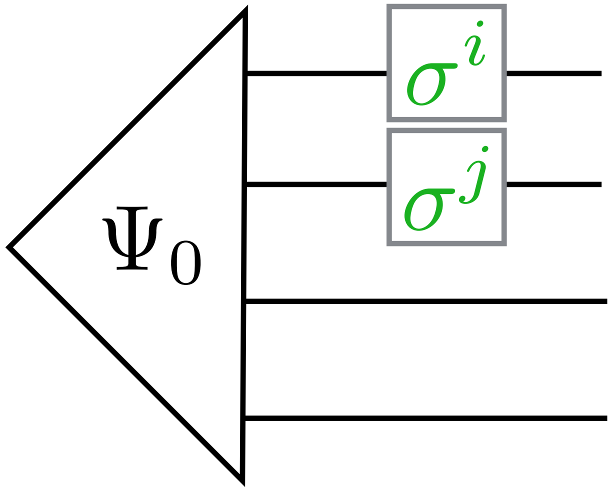

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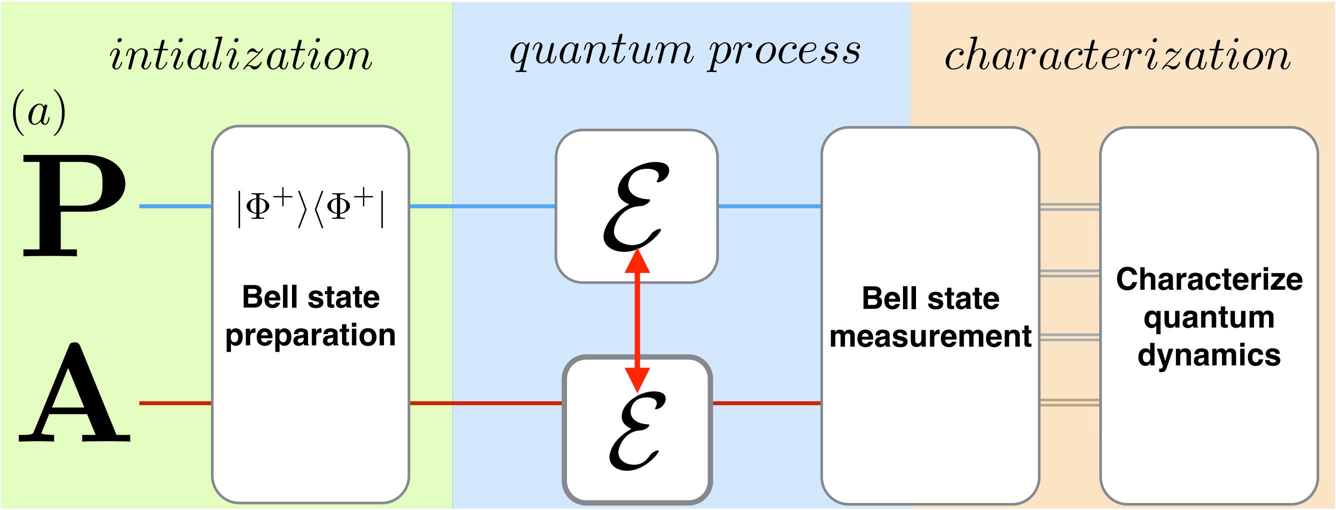

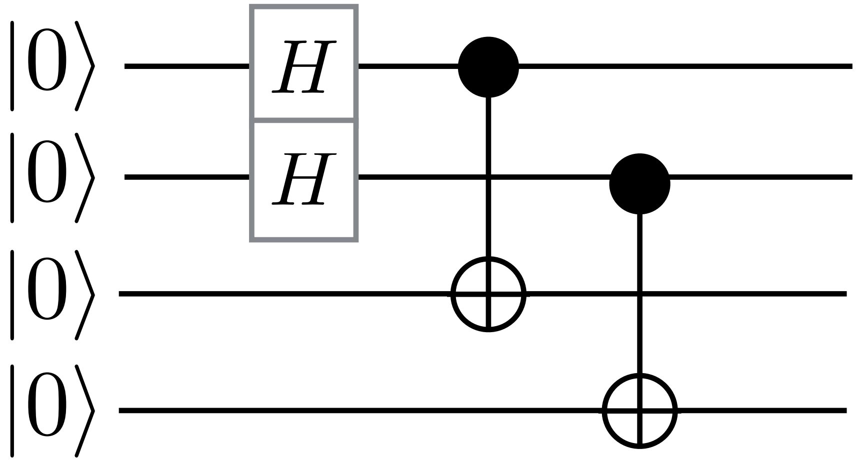

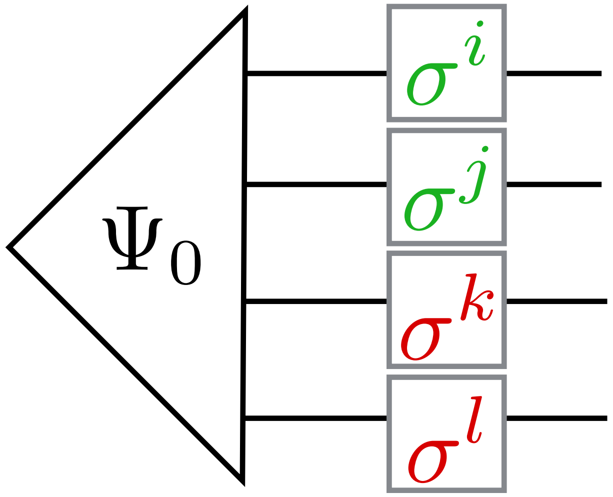

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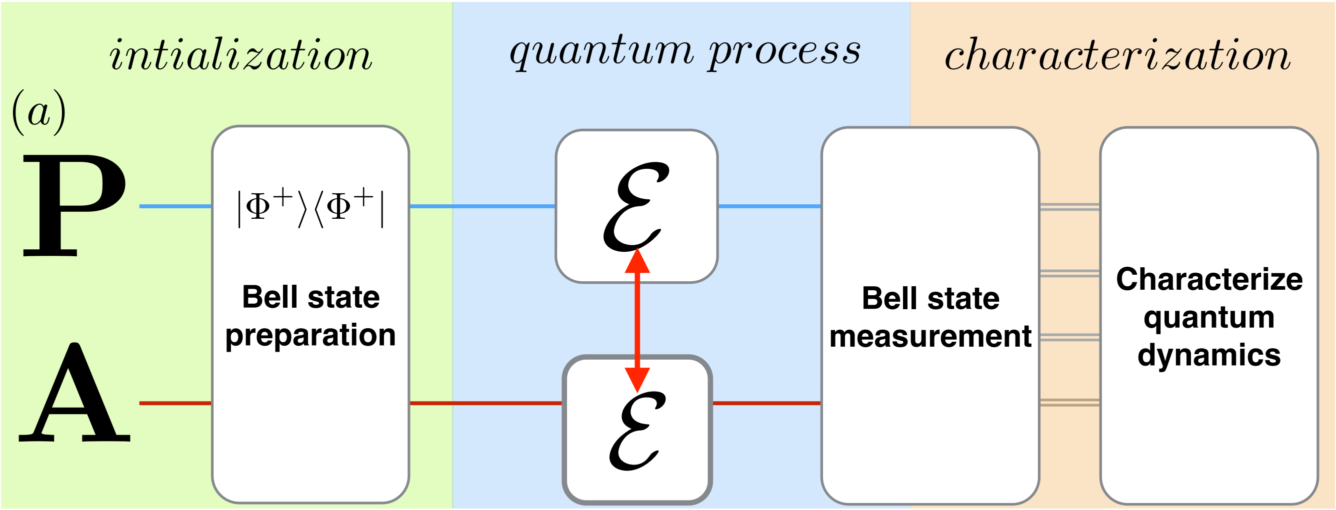

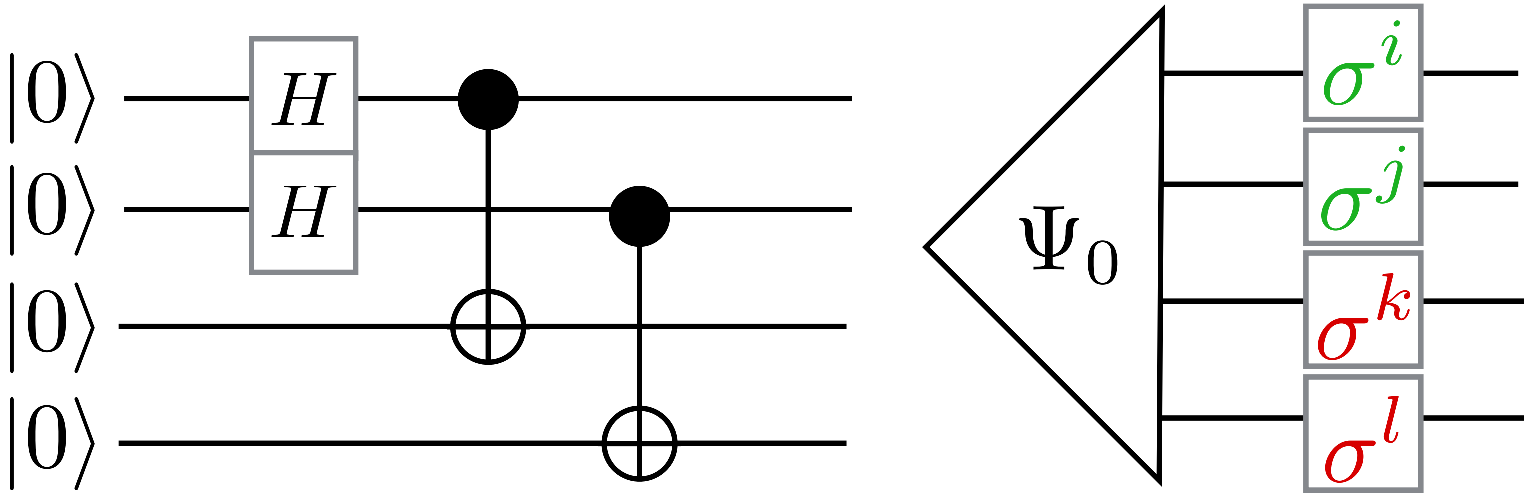

Code invariant under

$\mathbf{A} \Leftrightarrow \mathbf{P}$

Entanglement Assisted Codes

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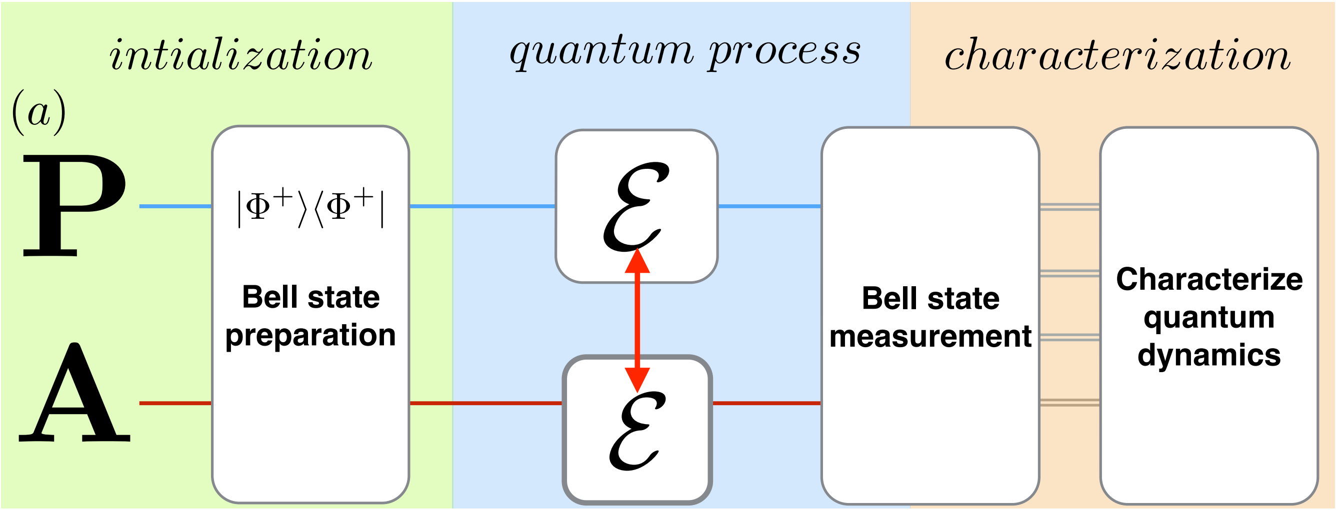
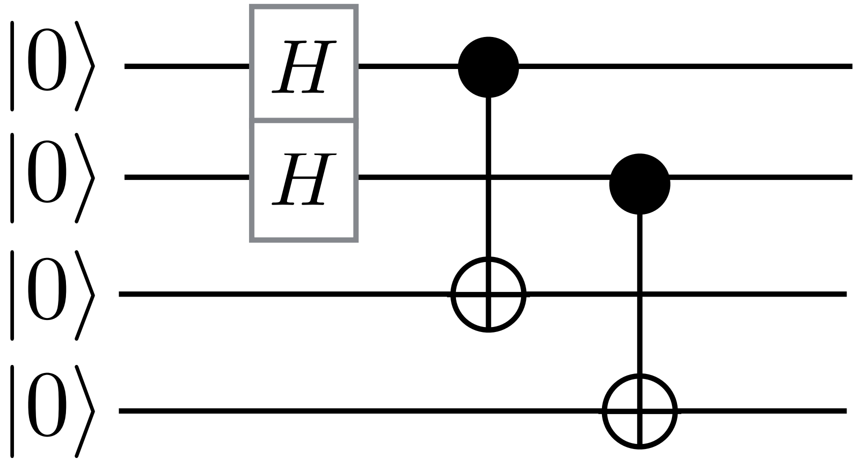
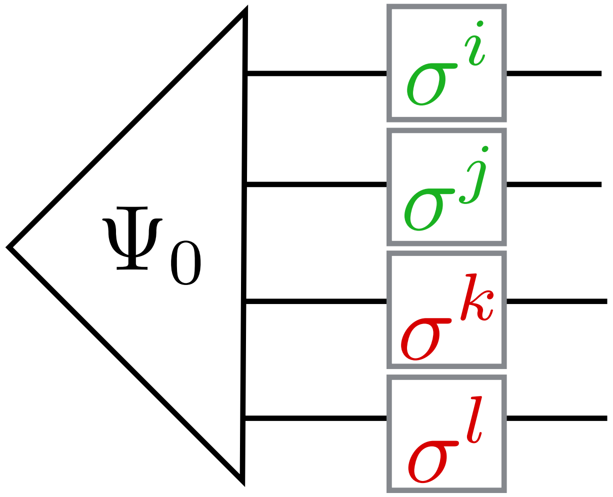






Code invariant under

$\mathbf{A} \Leftrightarrow \mathbf{P}$

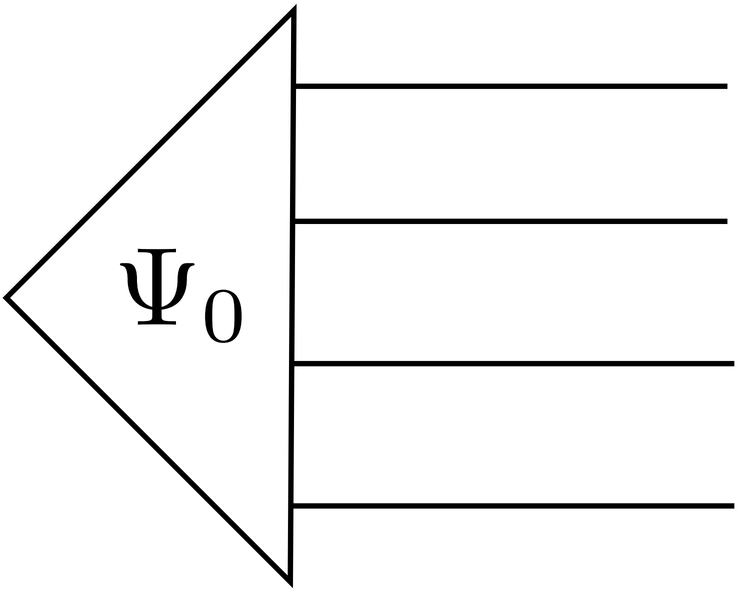
Goal: Retain detection capabilities on **P** while differentiating between **A** and **P**

Solution: Expand Hilbert space by adding ancilla qubits

Asymmetric Code

Asymmetric Code

$\mathcal{S}_0 = \langle XIXI, IXIX, ZIZI, IZIZ \rangle$



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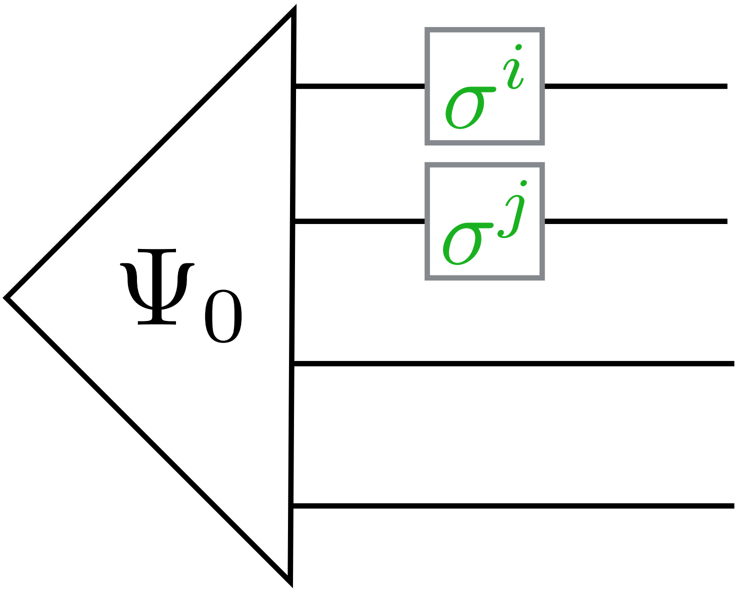

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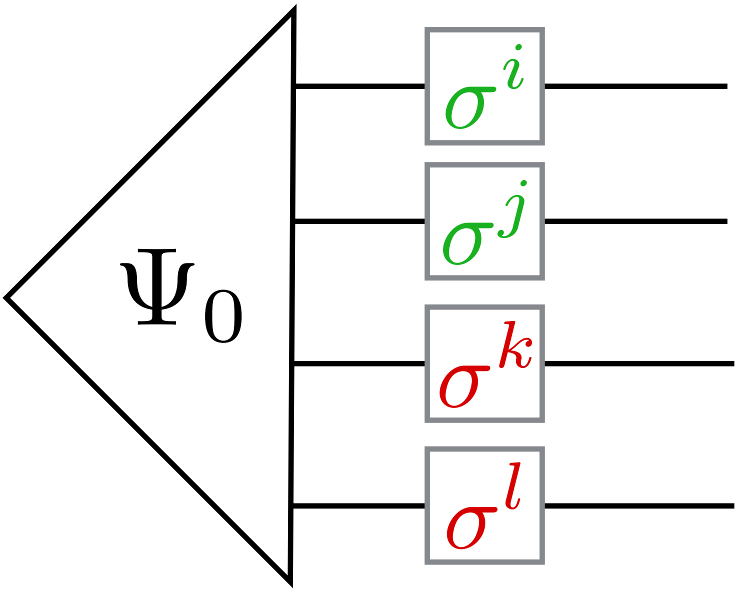
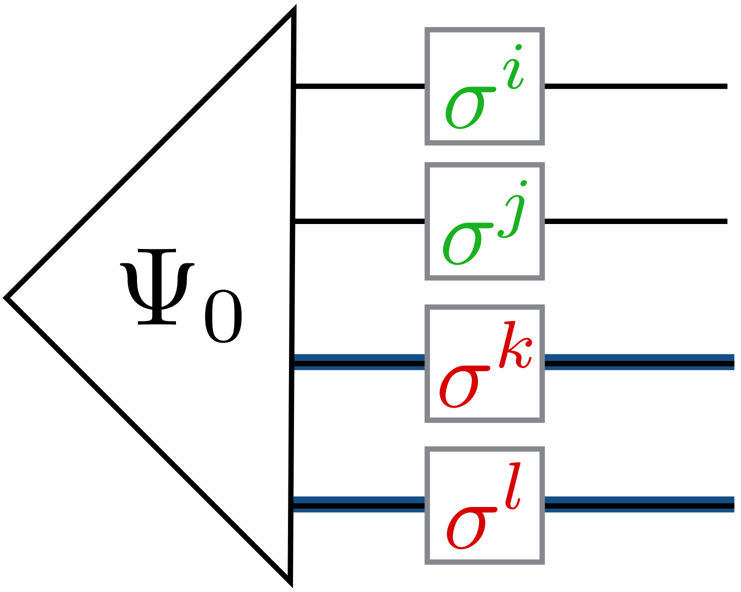


Asymmetric Code

$$\mathcal{S}_0 = \langle XI\overline{X}I, IXI\overline{X}, ZI\overline{Z}I, IZI\overline{Z}\rangle$$



[[4,2,2]] Encoding

$$S_4 = \langle XXXX, ZZZZ\rangle$$

$$\xrightarrow{\hspace{4em}}$$

$$\overline{X}_1 = XXII \quad \overline{Z}_1 = ZIZI$$

$$\overline{X}_2 = IXIX \quad \overline{Z}_2 = IIZZ$$

Asymmetric Code

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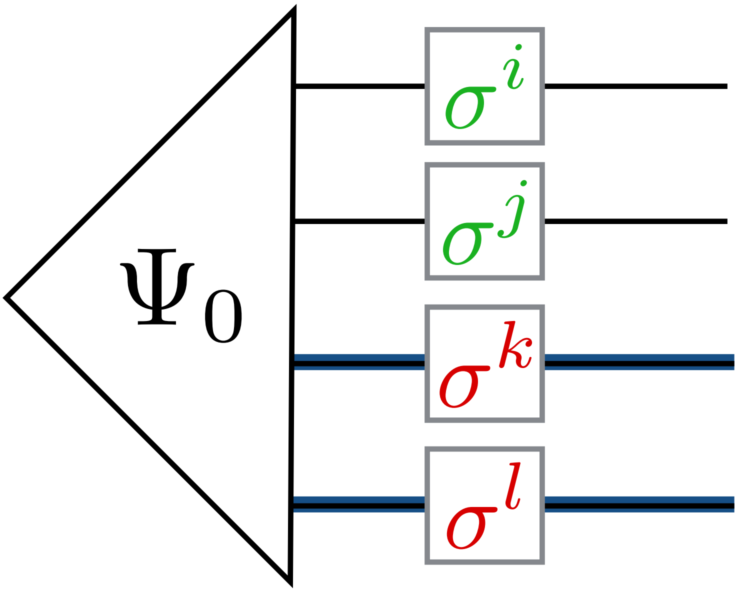
$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII,$$
$$ZIZIZI, IXIXIX, IZIIZZ\rangle.$$



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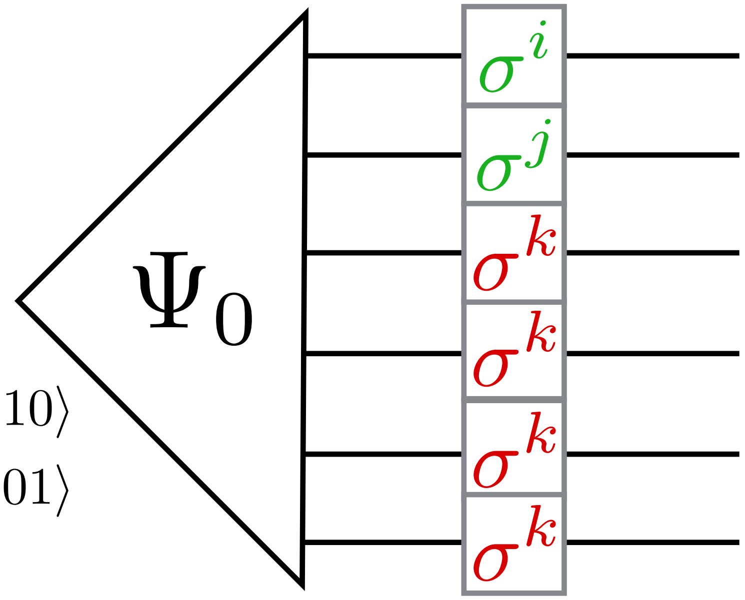
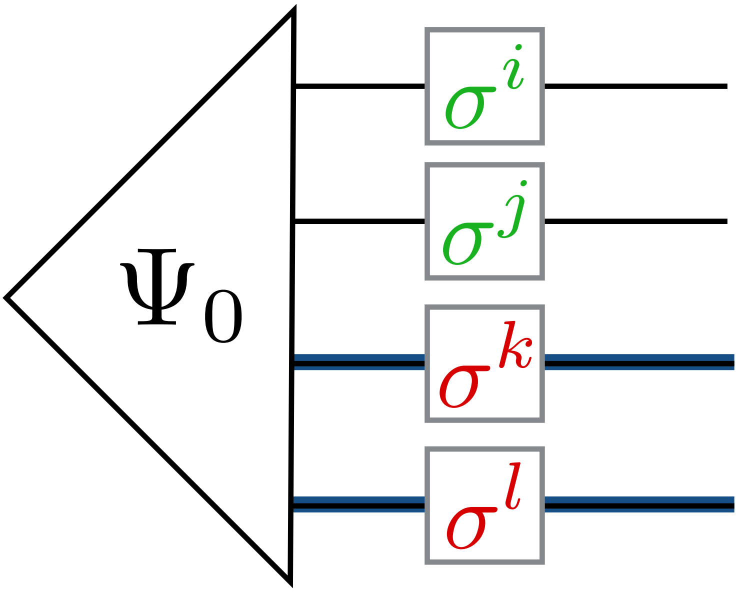


Asymmetric Code

$$\mathcal{S}_0 = \langle XI\bar{X}I, IXI\bar{X}, ZI\bar{Z}I, IZI\bar{Z} \rangle$$

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, ZIZIZI, IXIXIX, IZIIZZ \rangle.$$



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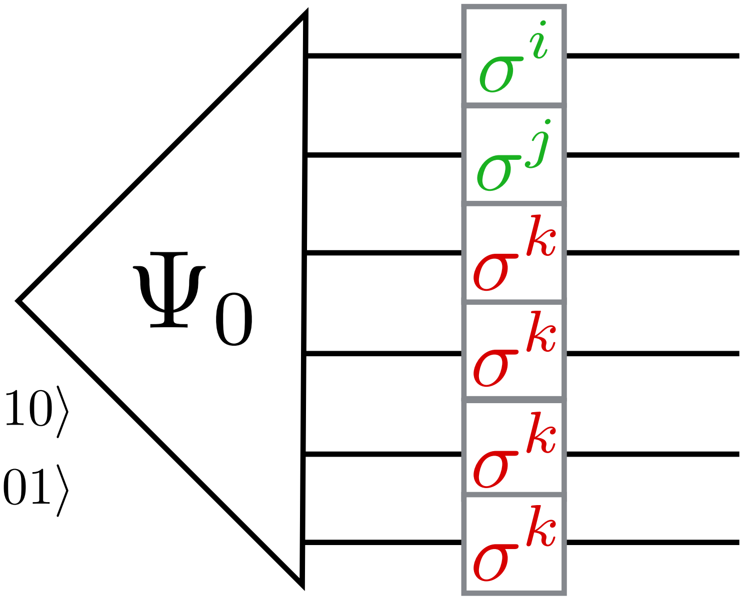

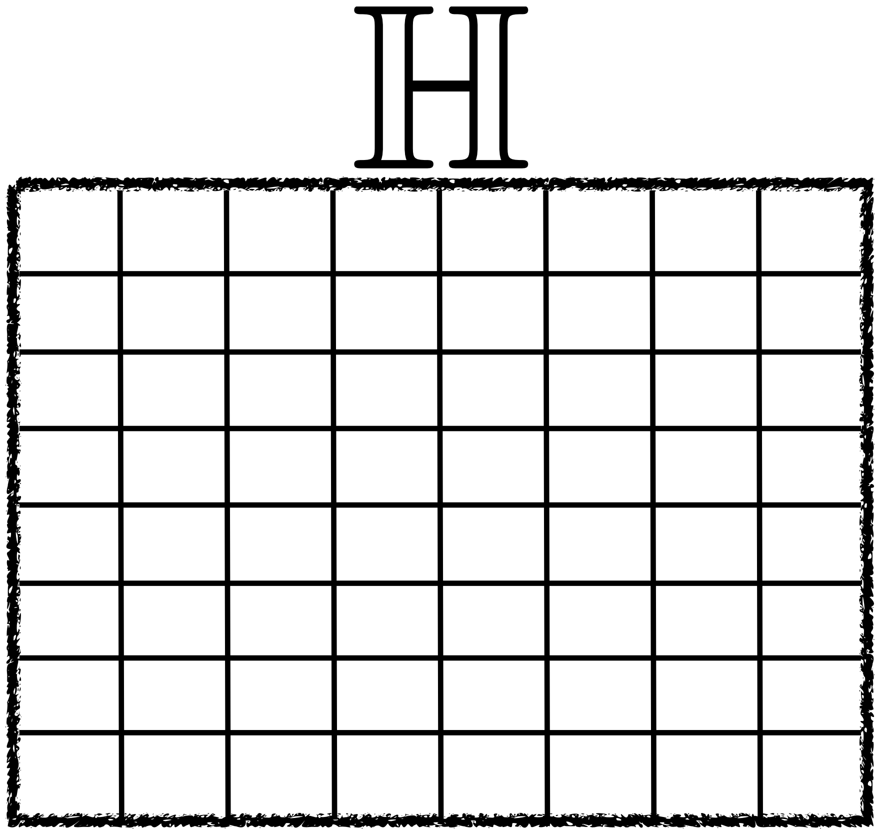

Asymmetric Code

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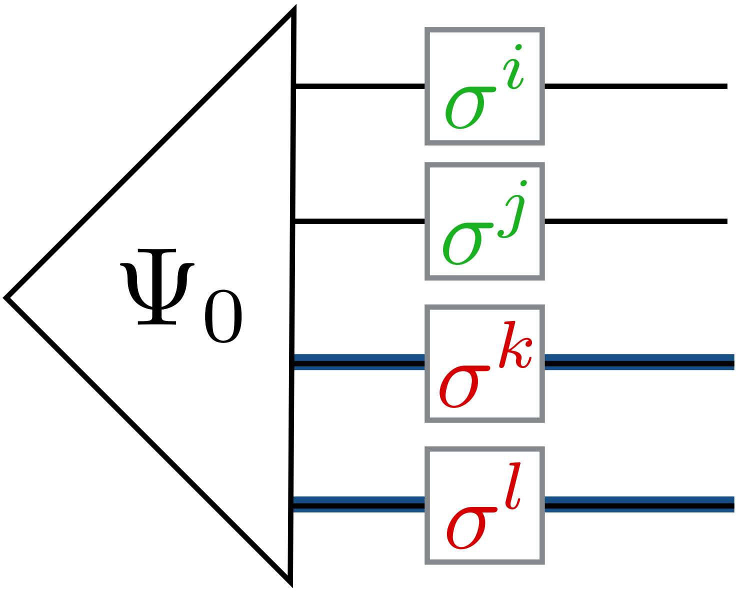

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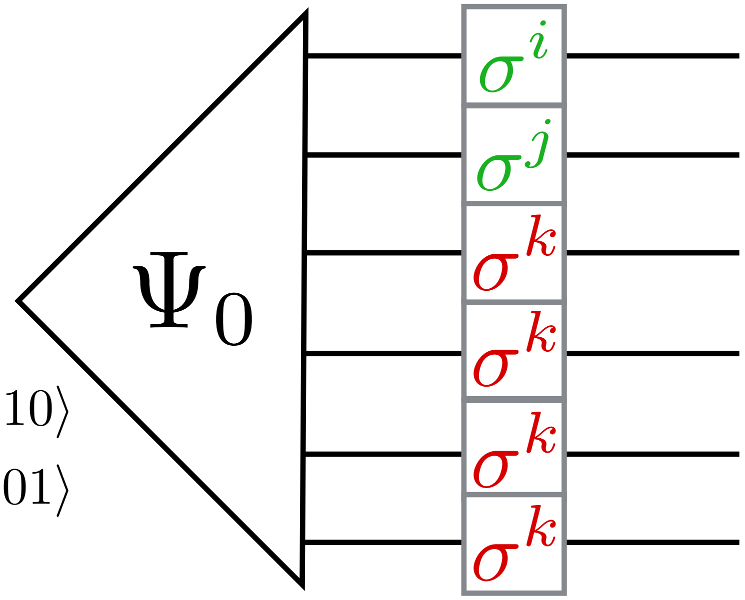
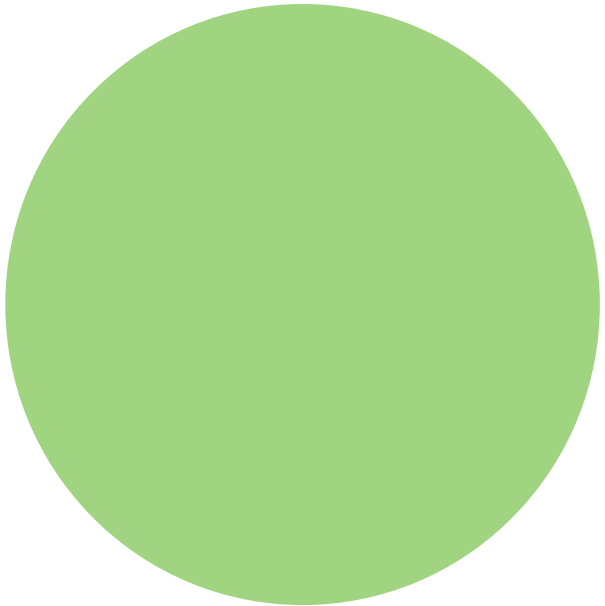
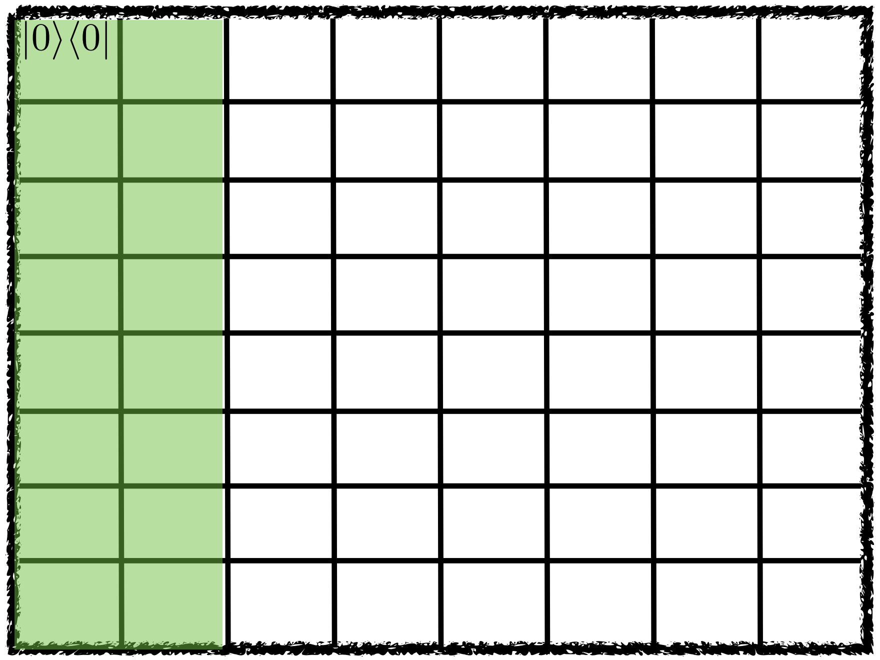


$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$



$\mathbb{H}$



Asymmetric Code

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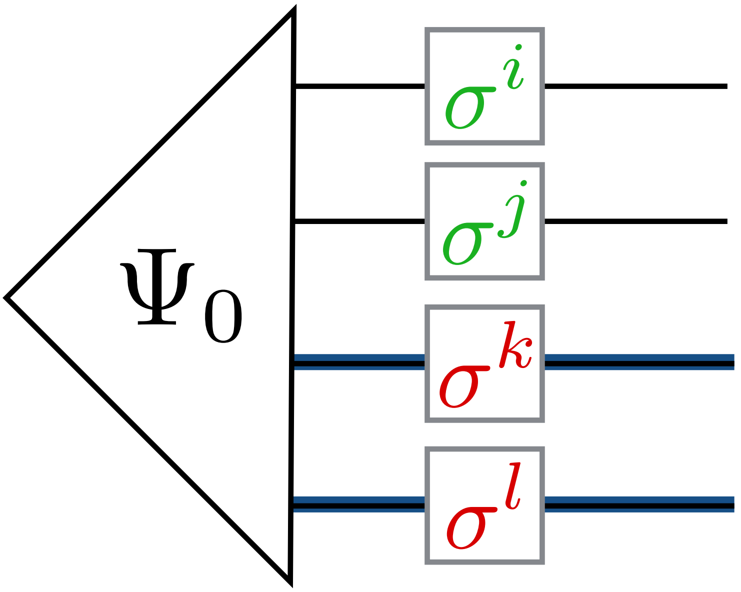

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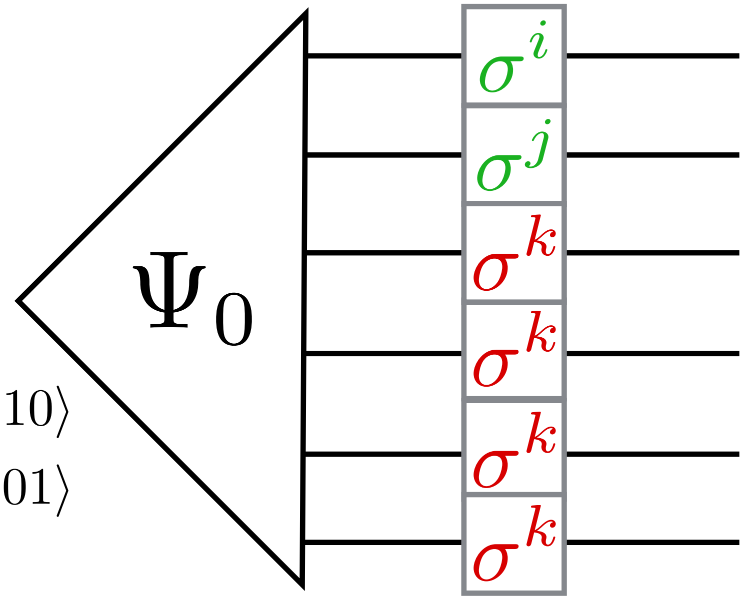

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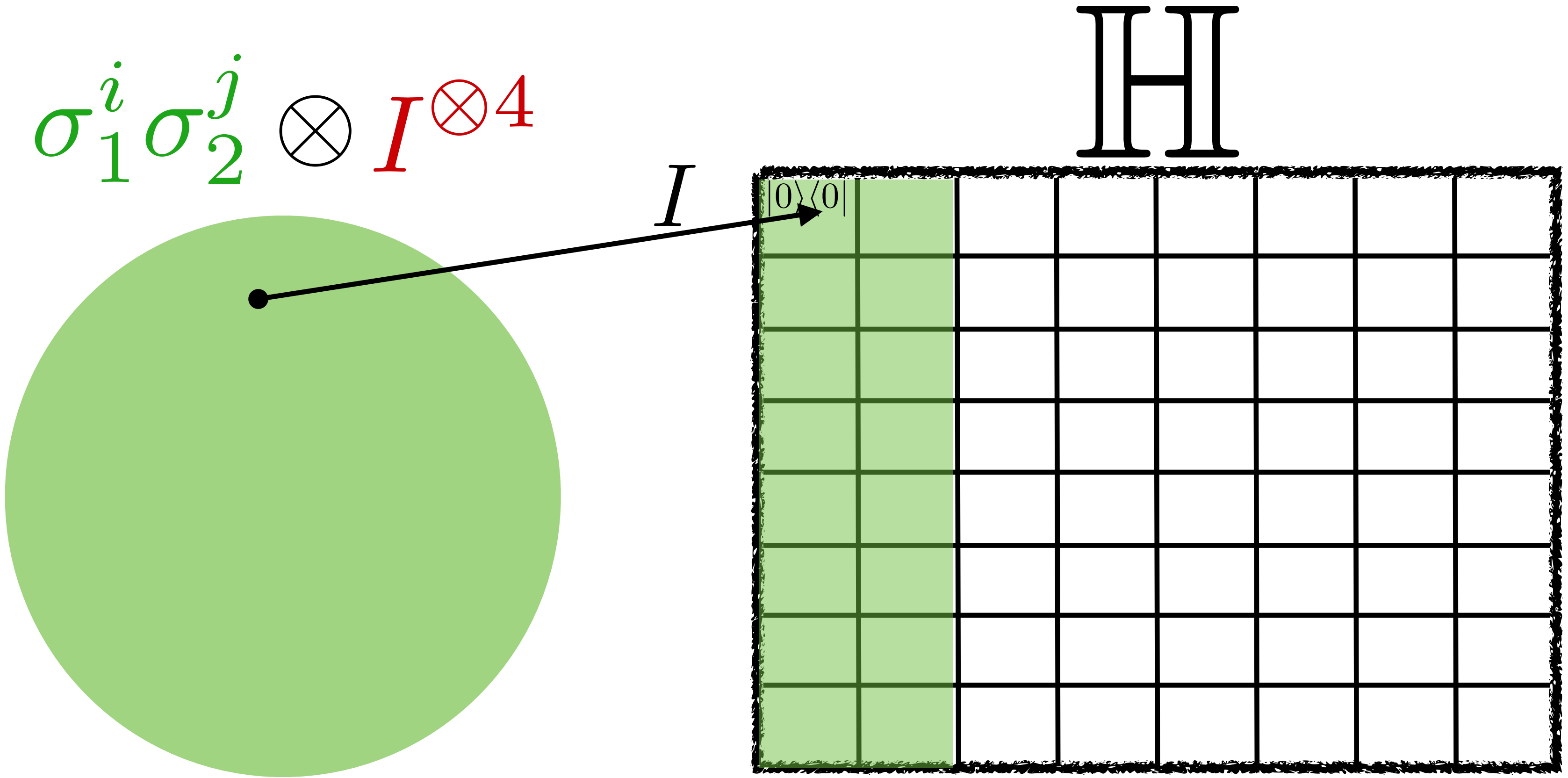

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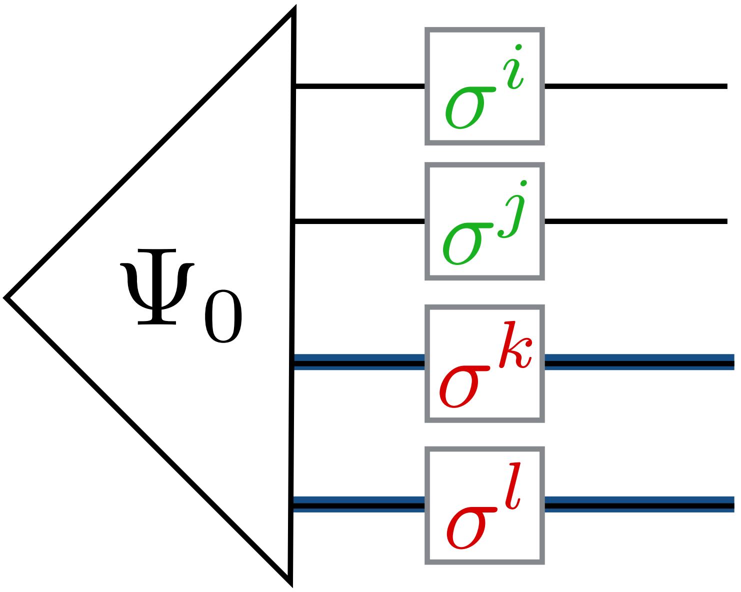

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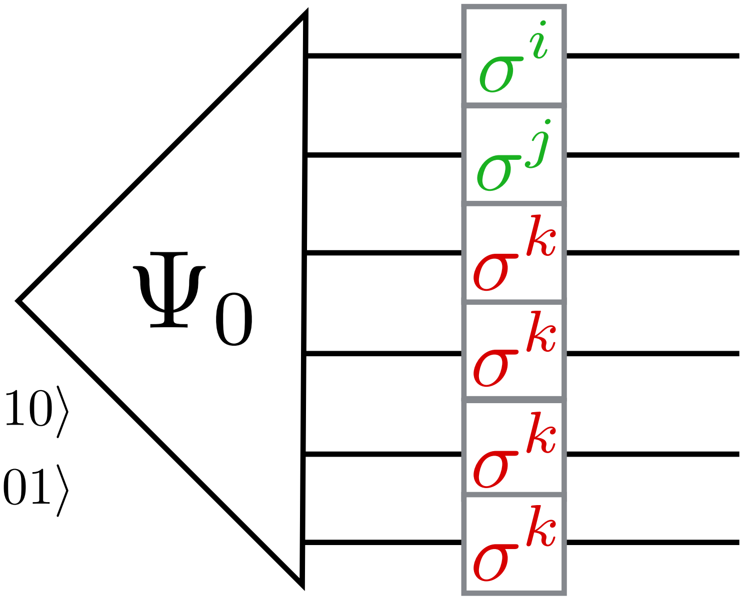

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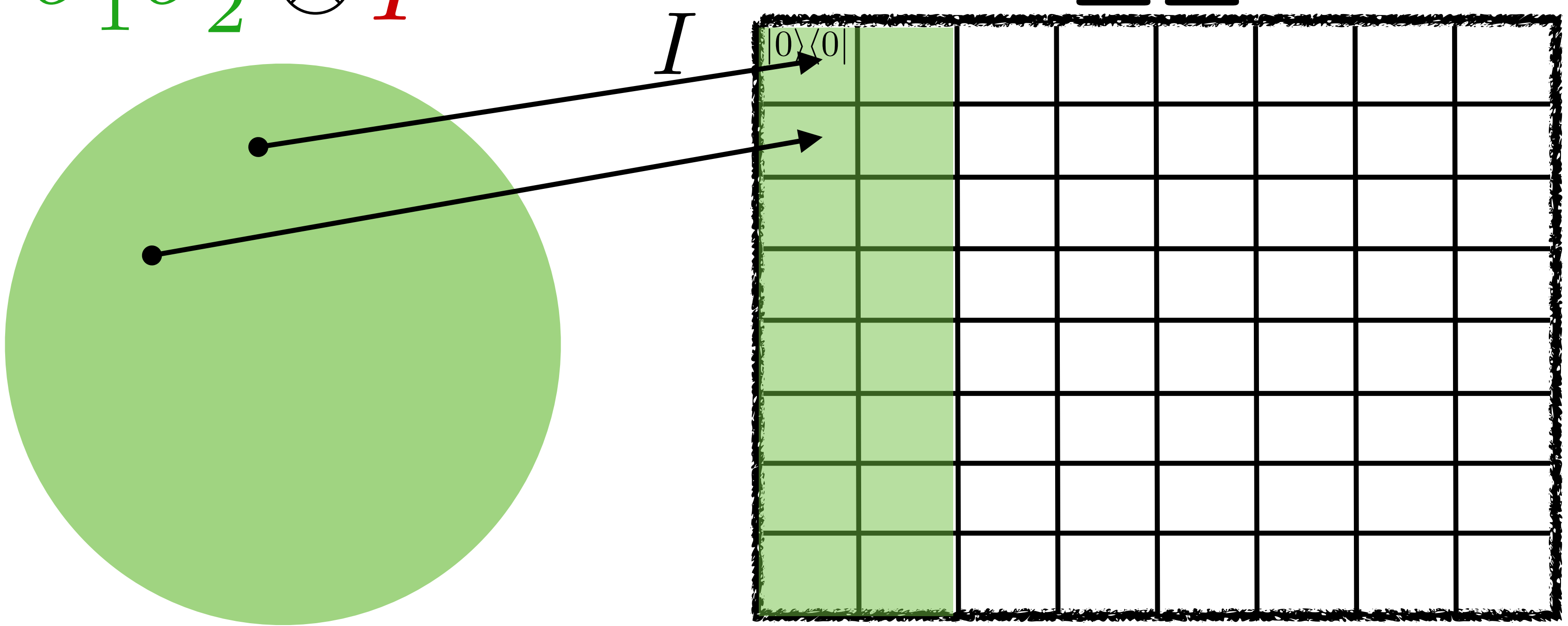

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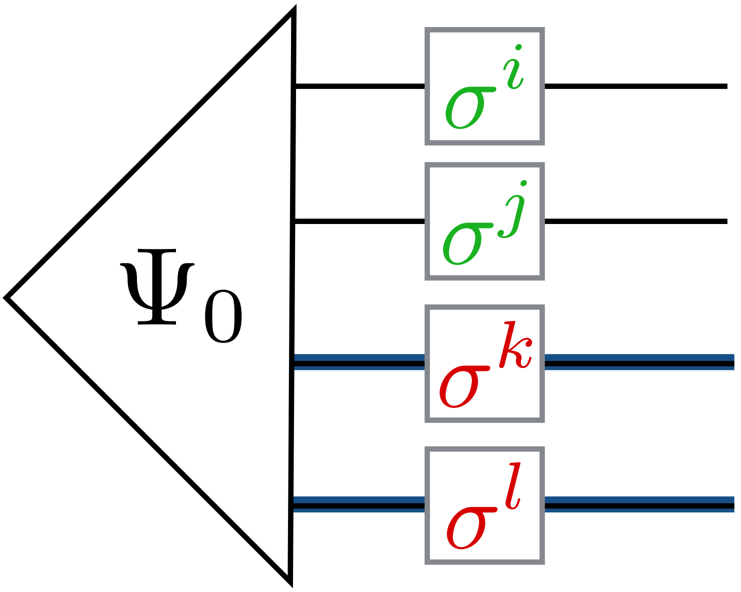

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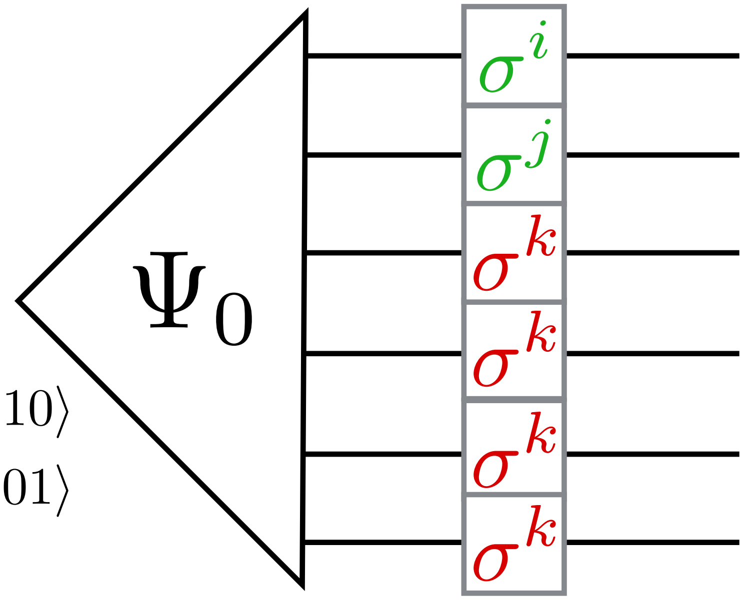

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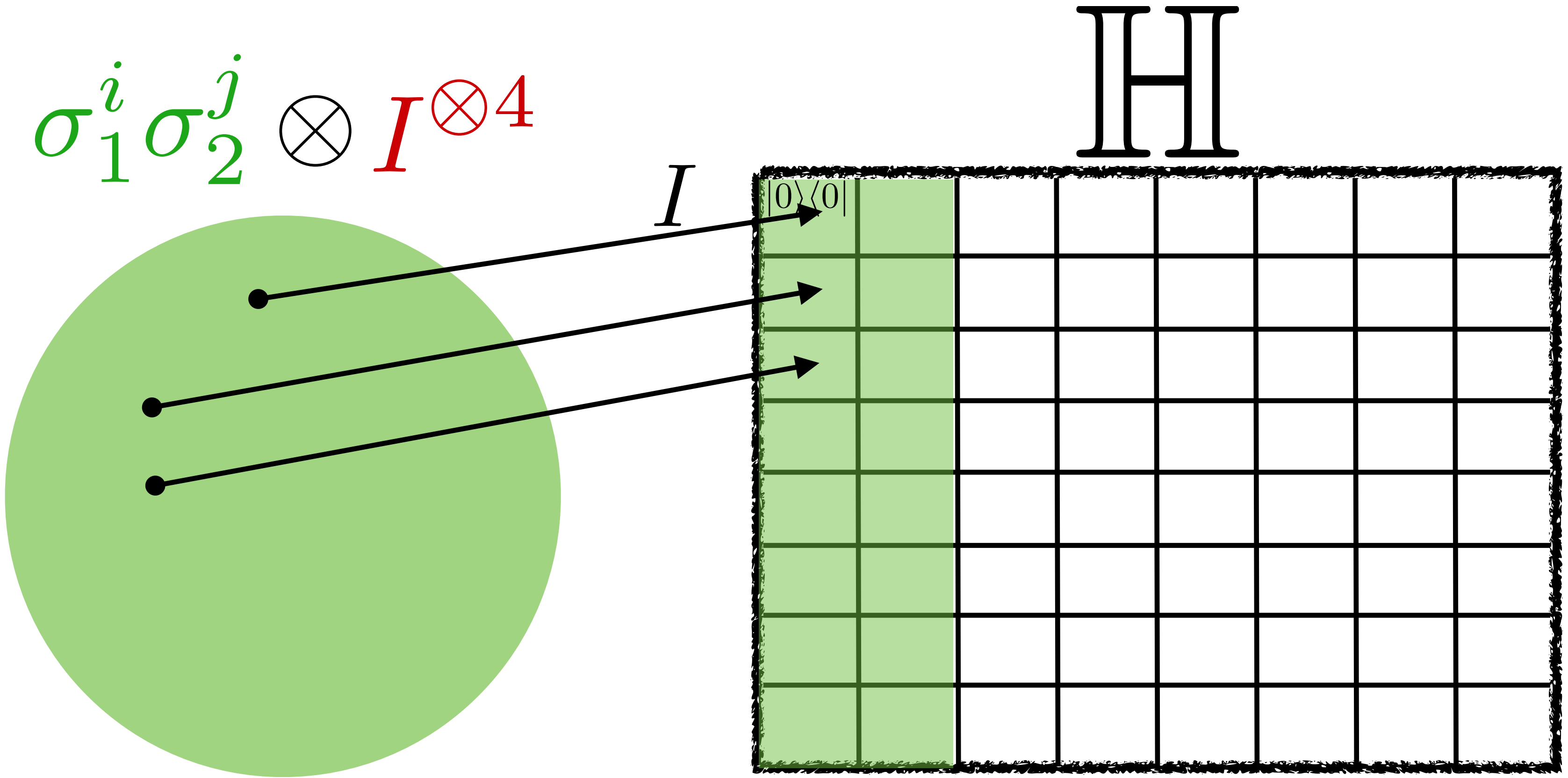

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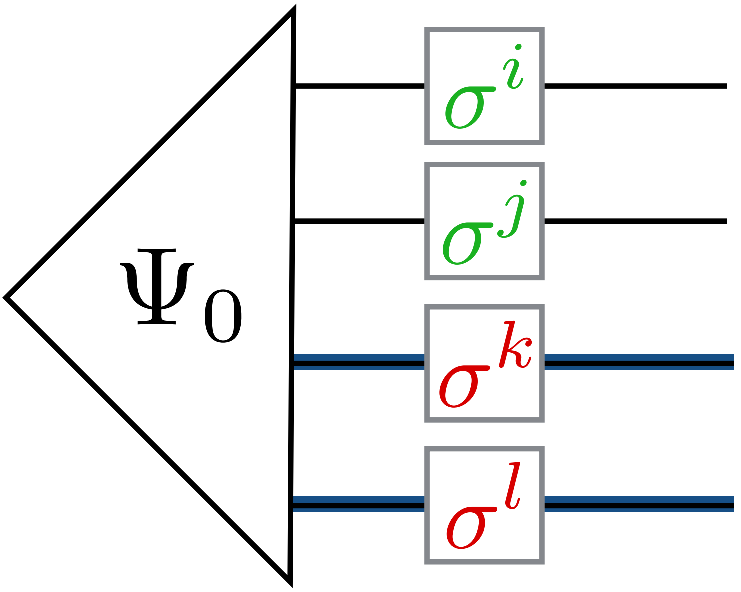

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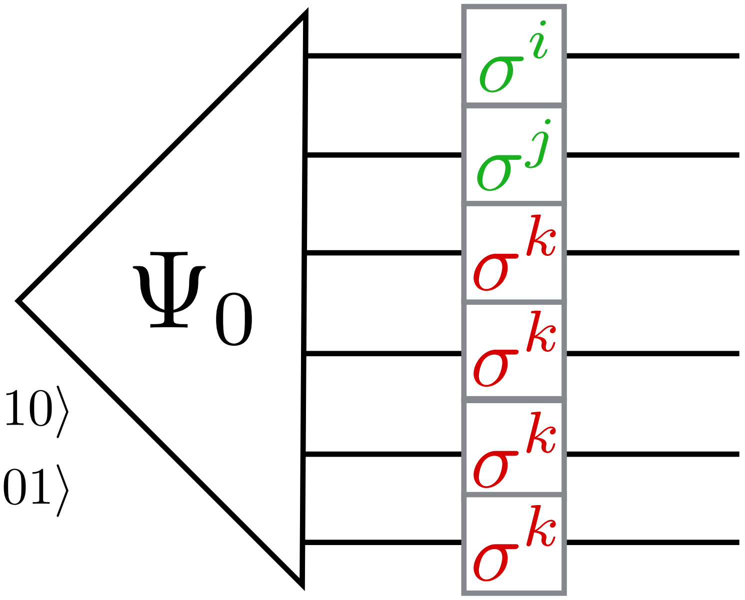

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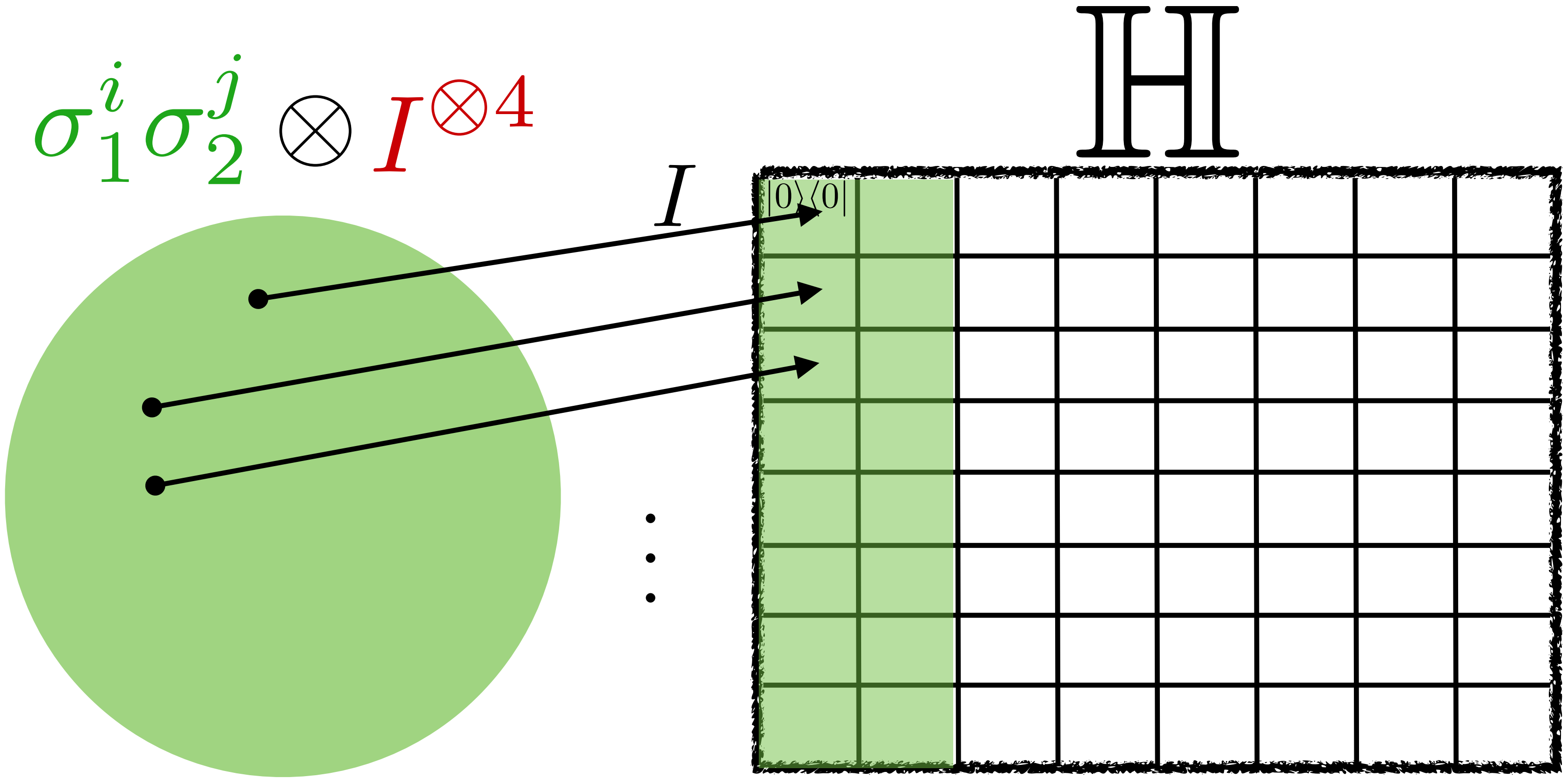

Asymmetric Code

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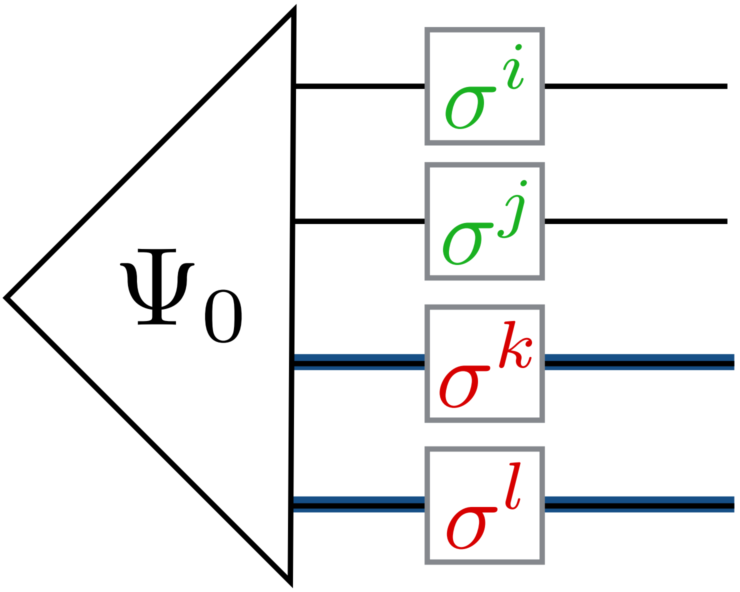

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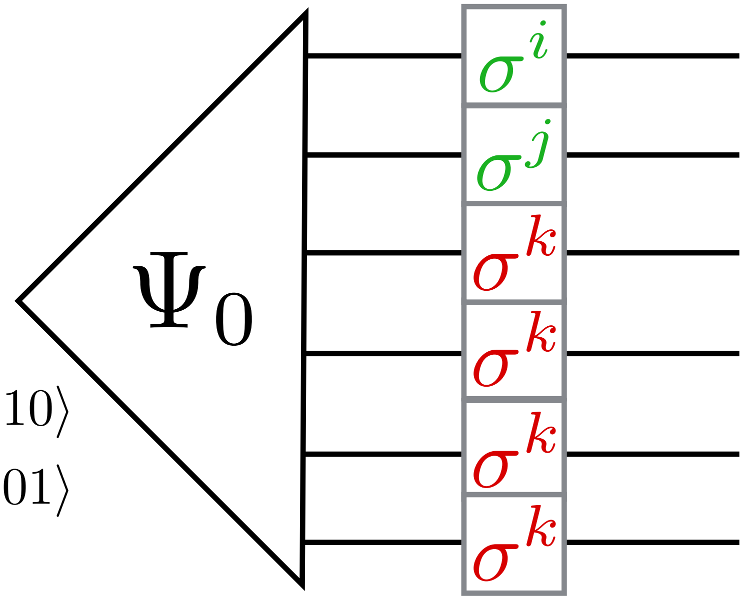

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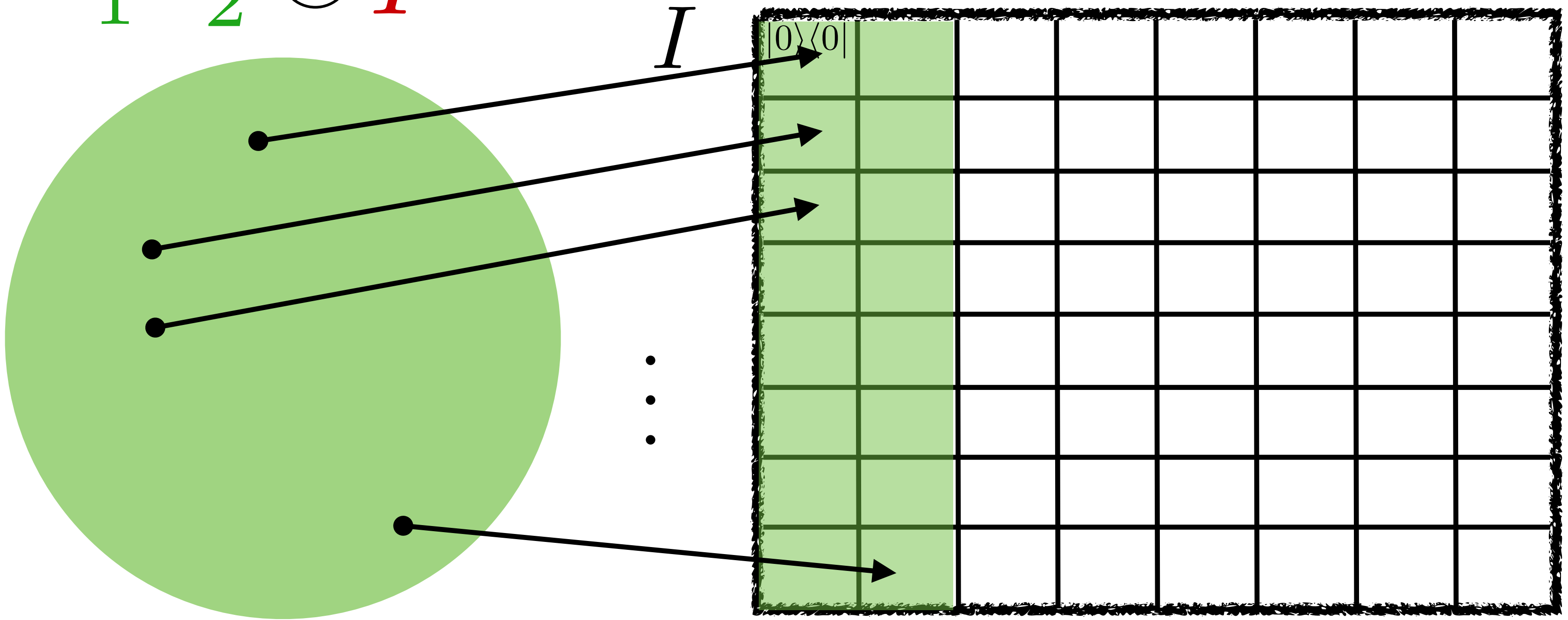
$\mathbb{H}$



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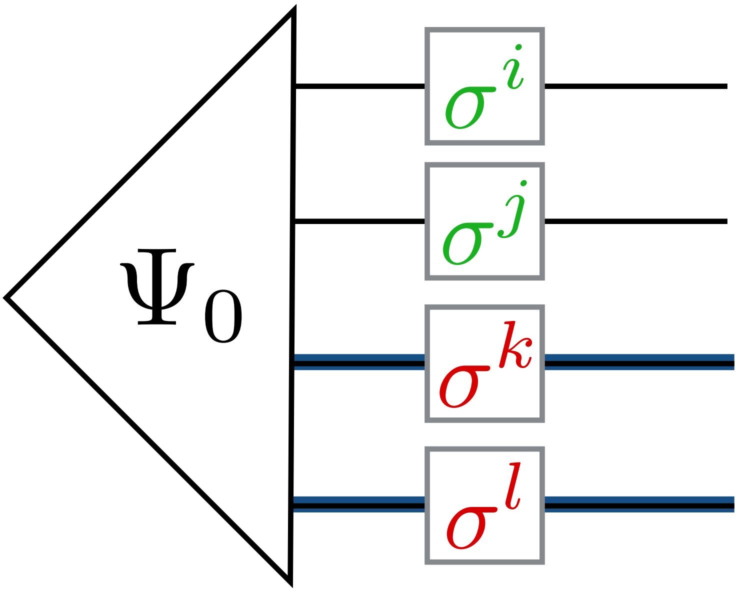

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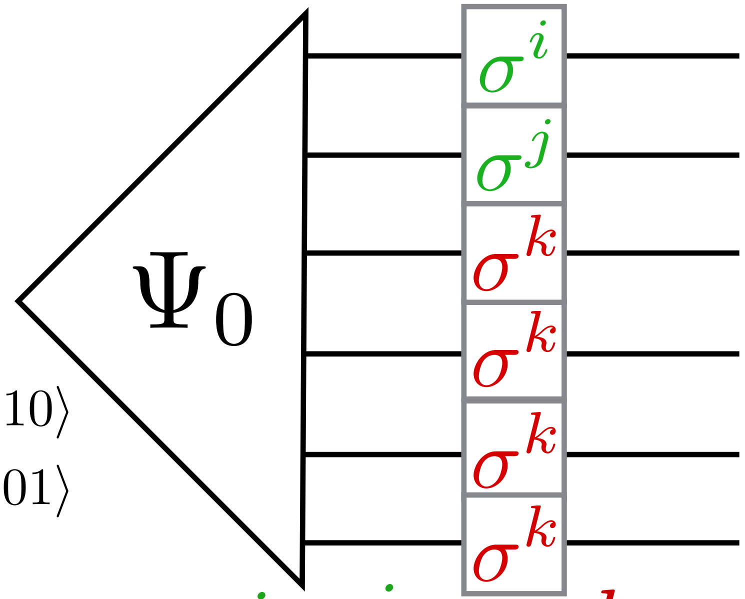
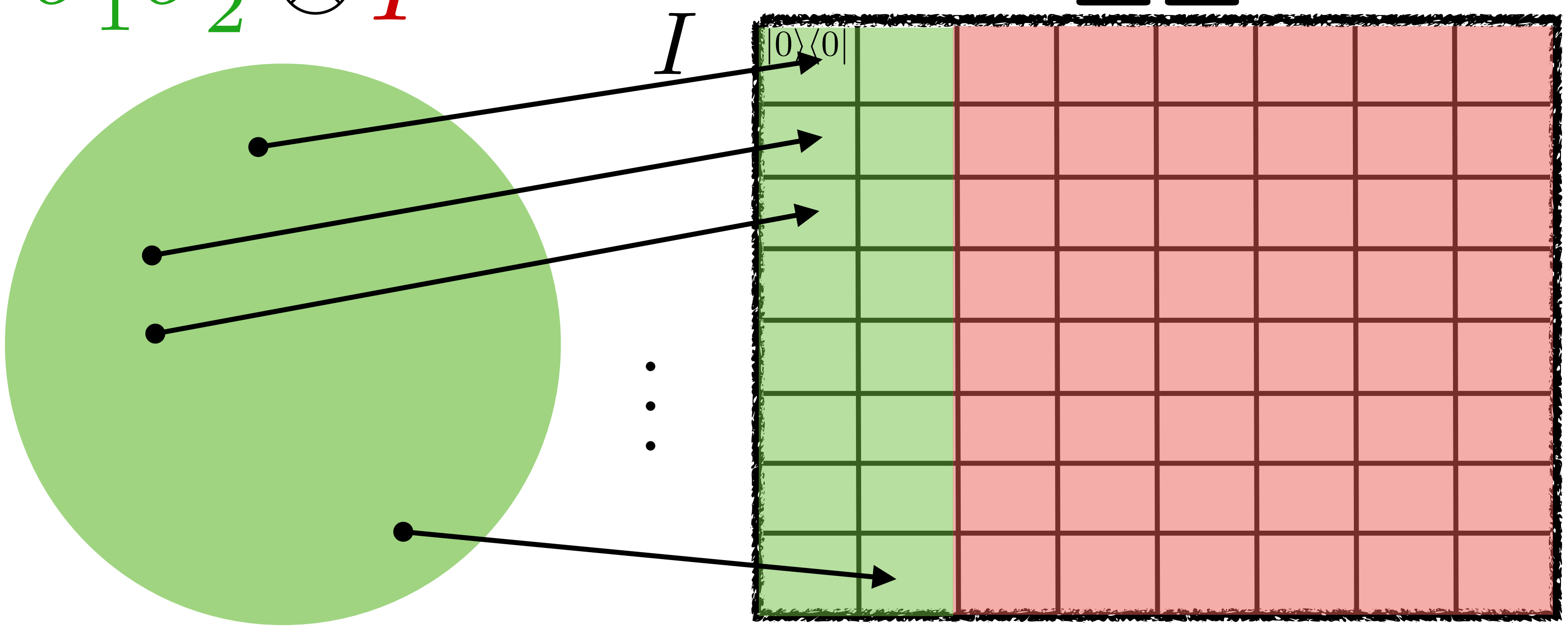
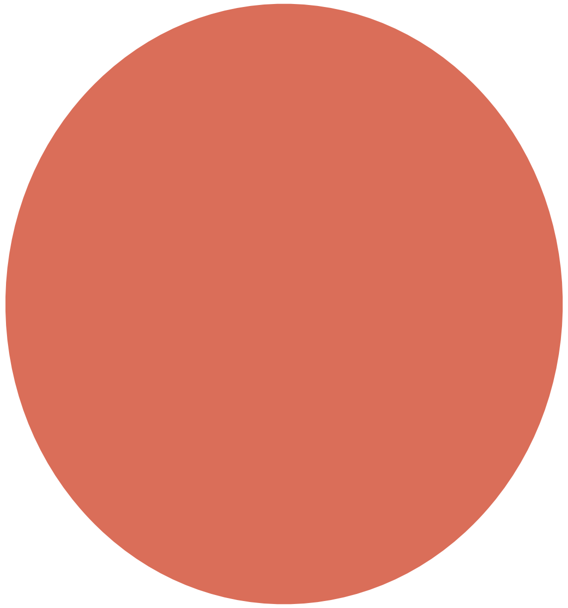


$\sigma_1^i \sigma_2^j \otimes I^{\otimes 4}$

$\mathbb{H}$



$\sigma_1^i \sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$



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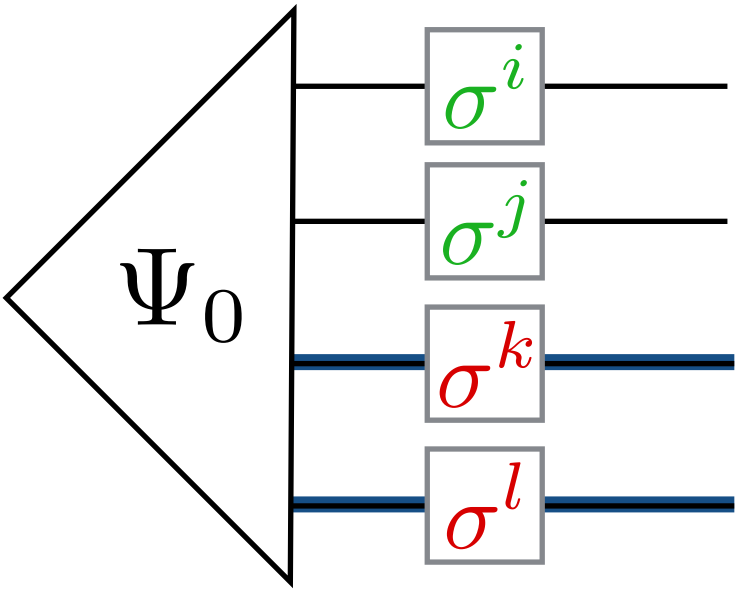

[[4,2,2]] Encoding

$S_4 = \langle XXXX, ZZZZ\rangle$

$$\overline{X}_1 = XXII \quad \overline{Z}_1 = ZIZI$$
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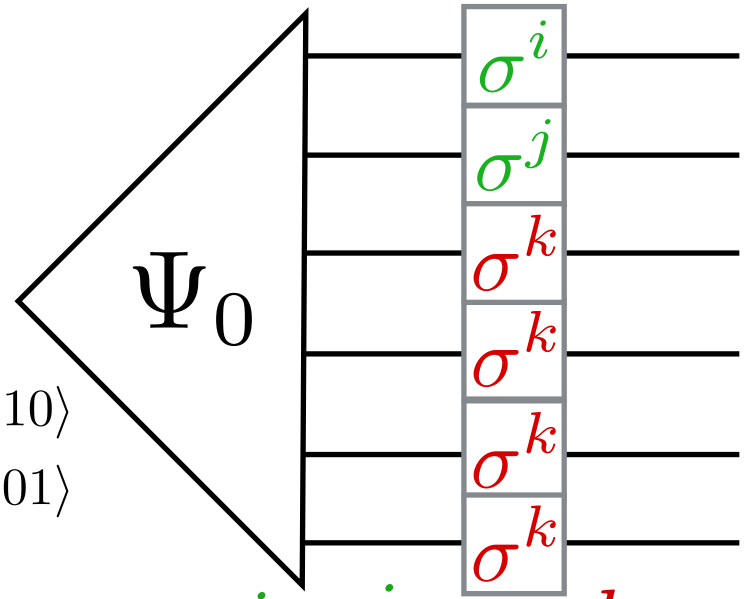
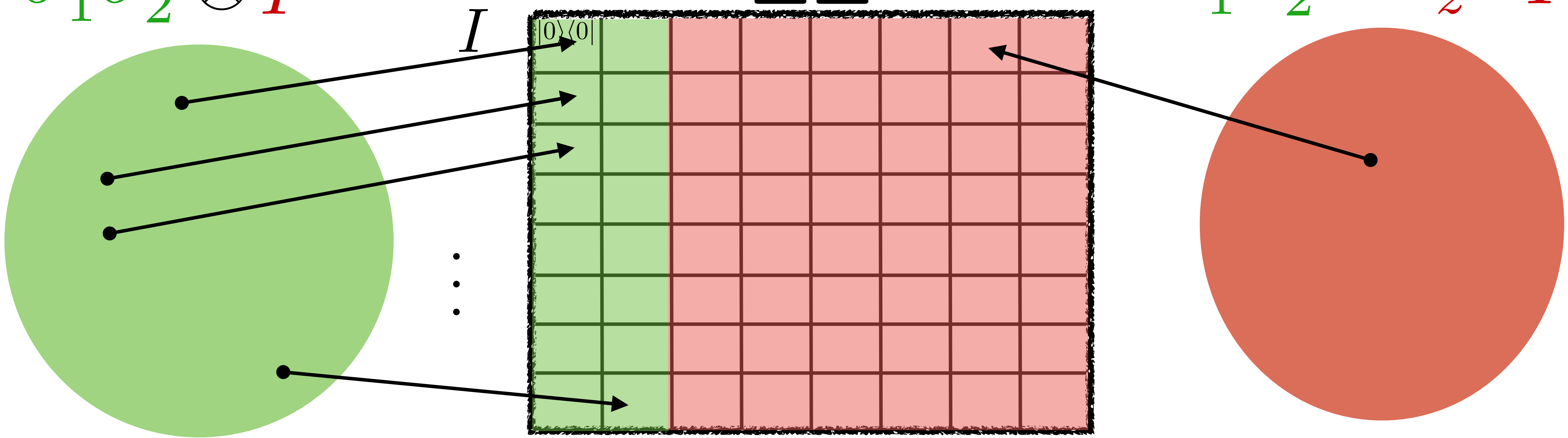


$\sigma_1^i\sigma_2^j \otimes I^{\otimes 4}$

$\mathbb{H}$

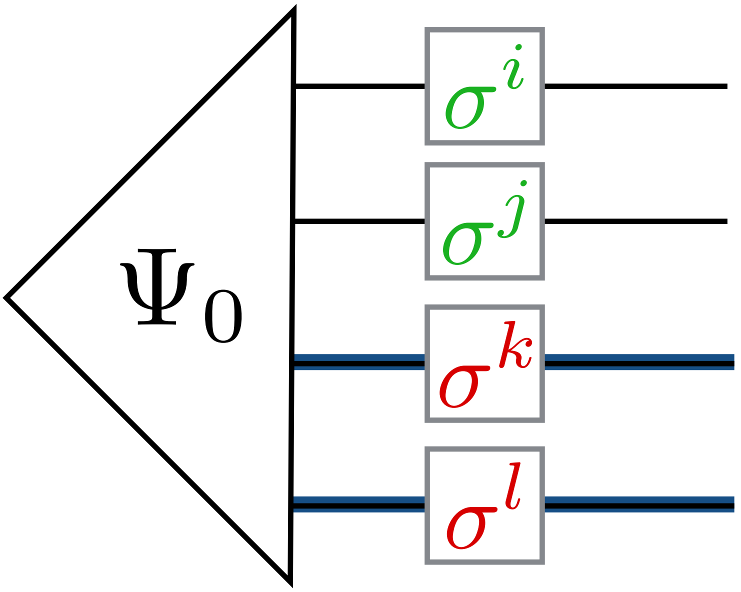
$\sigma_1^i\sigma_2^j \otimes \sigma_z^k \otimes I^{\otimes 3}$



Asymmetric Code

$$\mathcal{S}_0 = \langle XI\overline{X}I, IXI\overline{X}, ZI\overline{Z}I, IZI\overline{Z}\rangle$$

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, ZIZIZI, IXIXIX, IZIIZZ\rangle.$$



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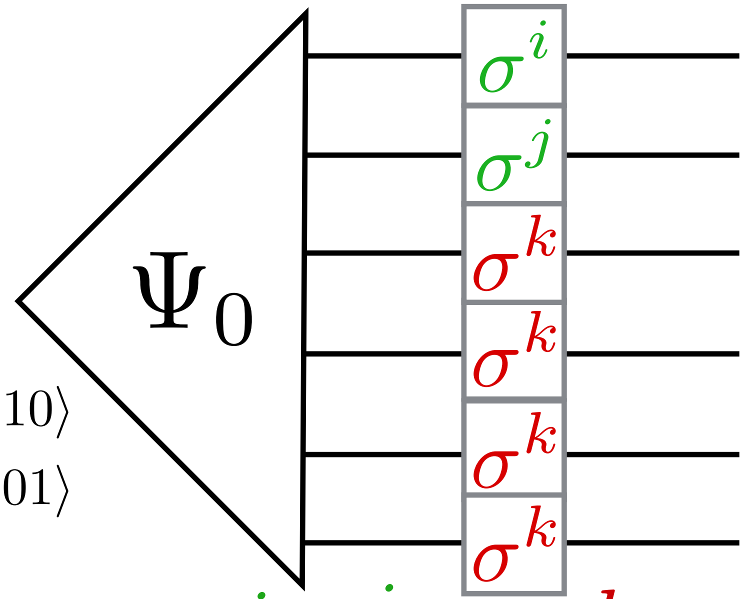

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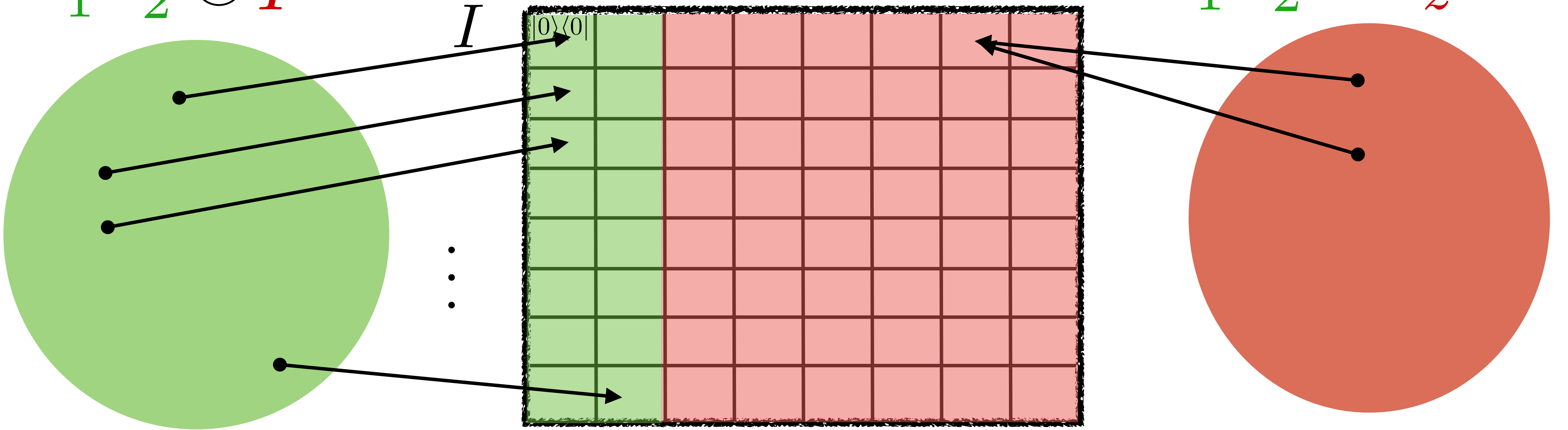

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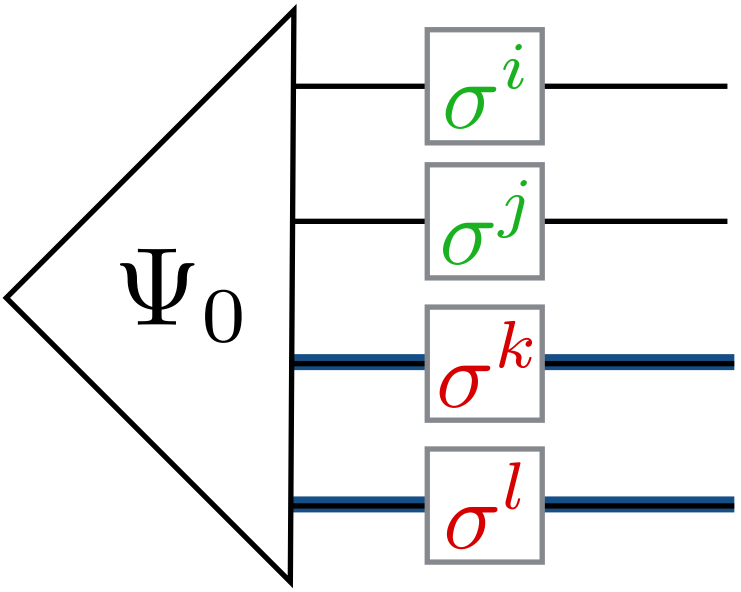

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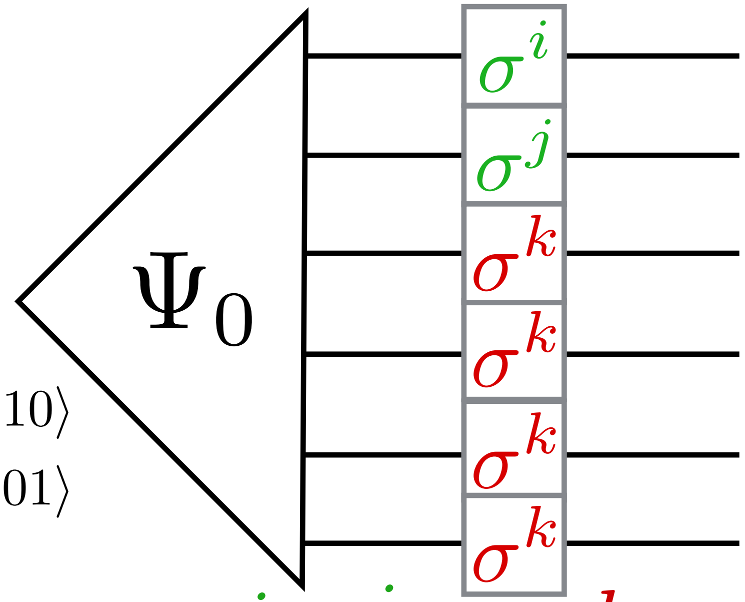

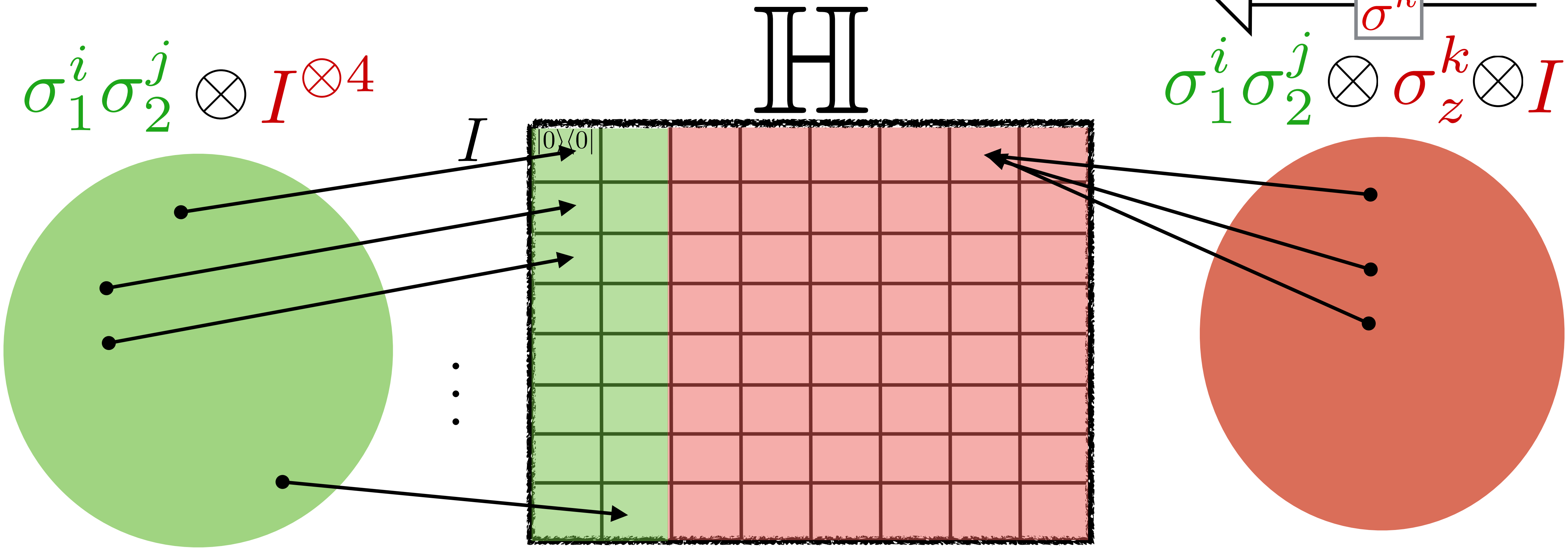

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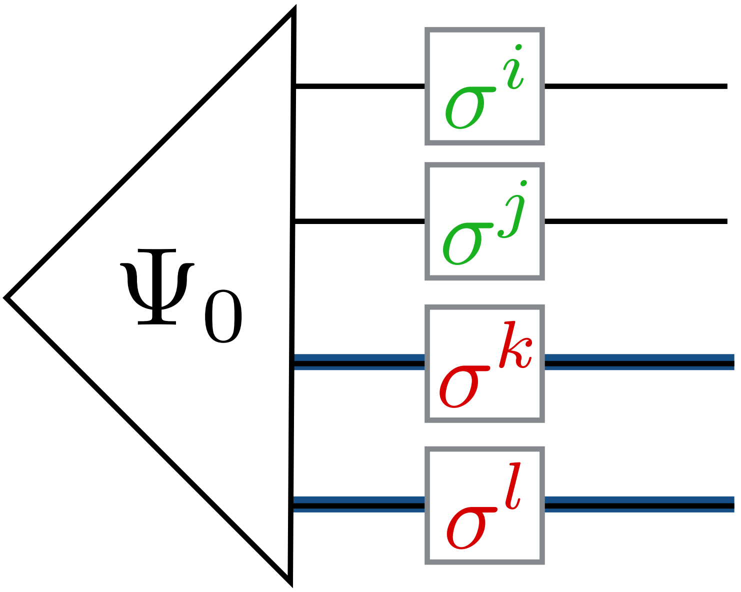

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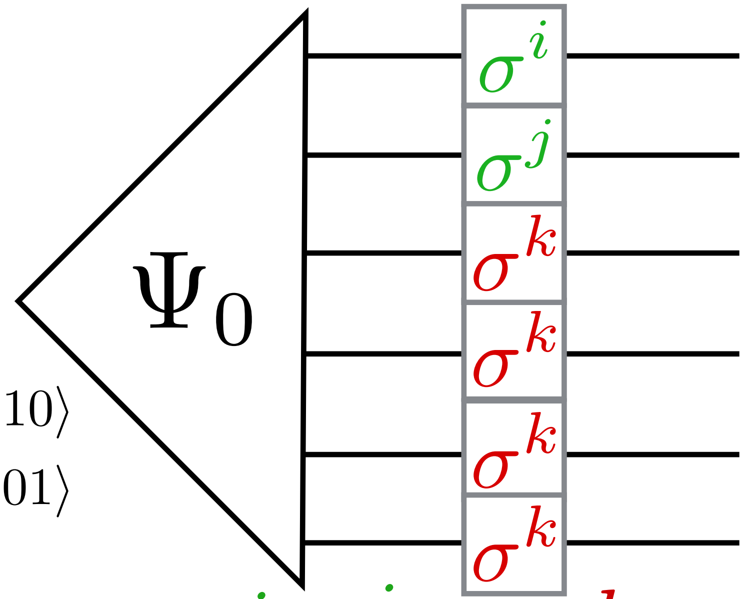

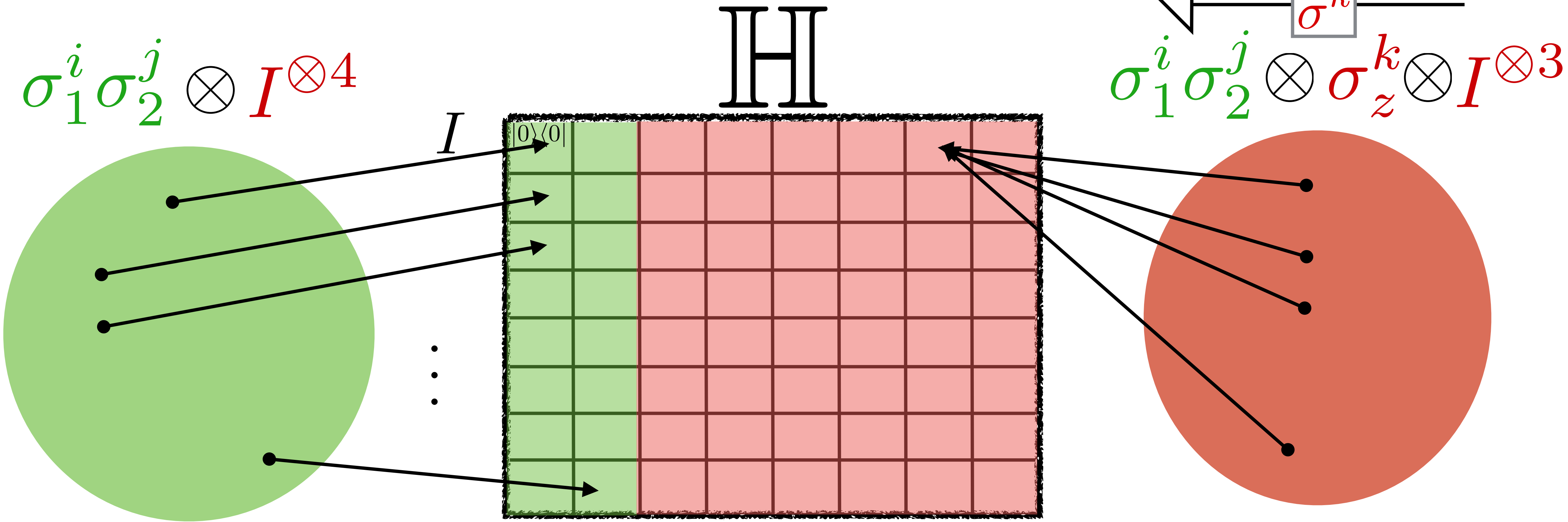

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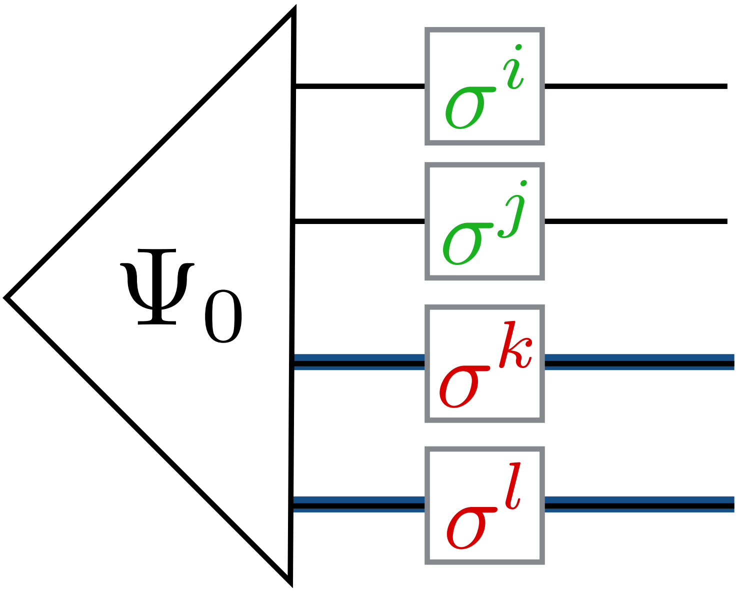

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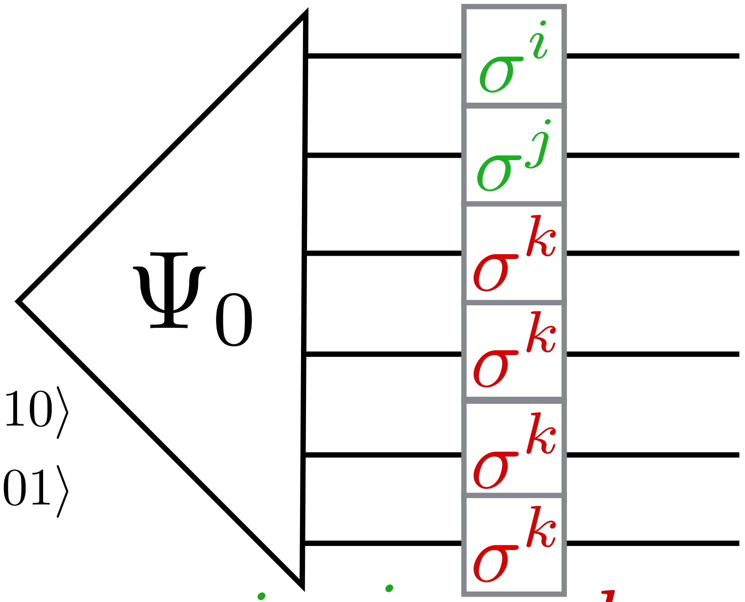

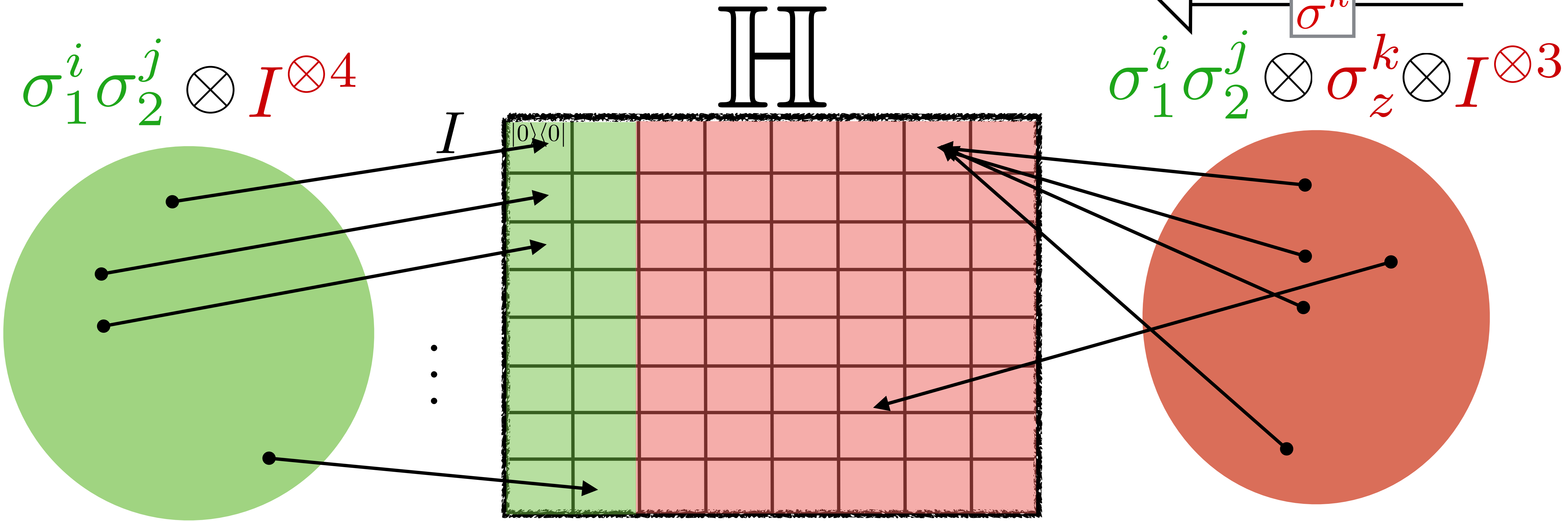

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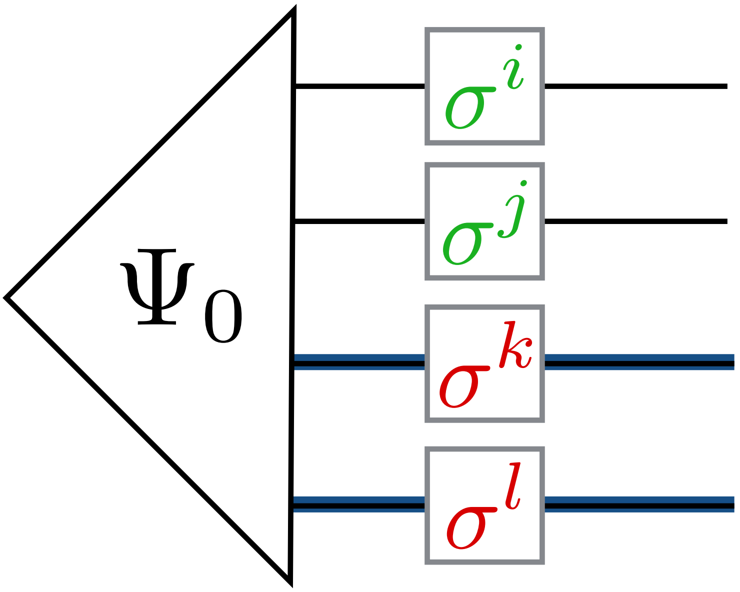

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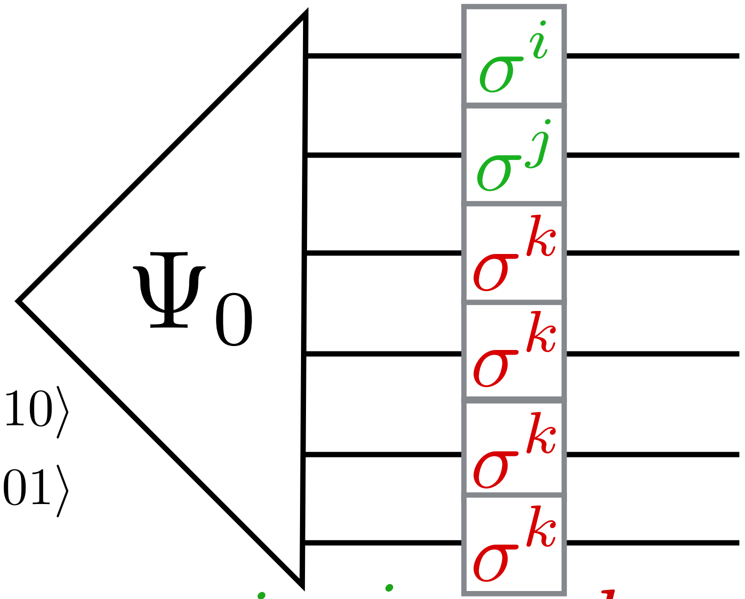

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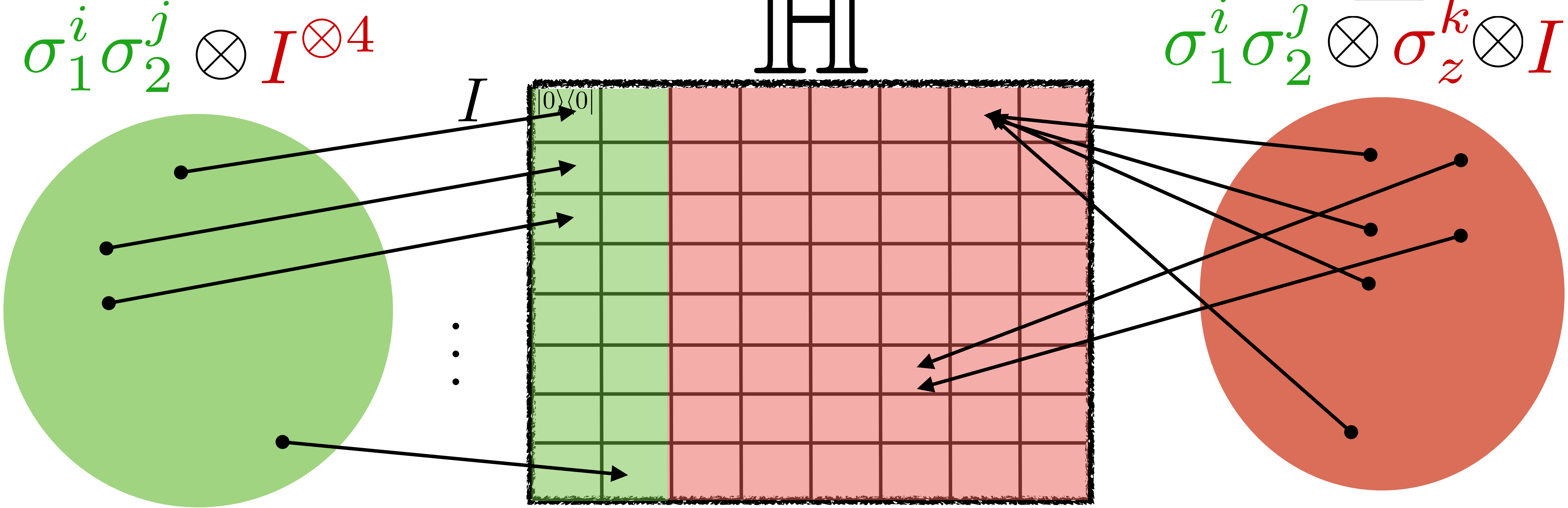

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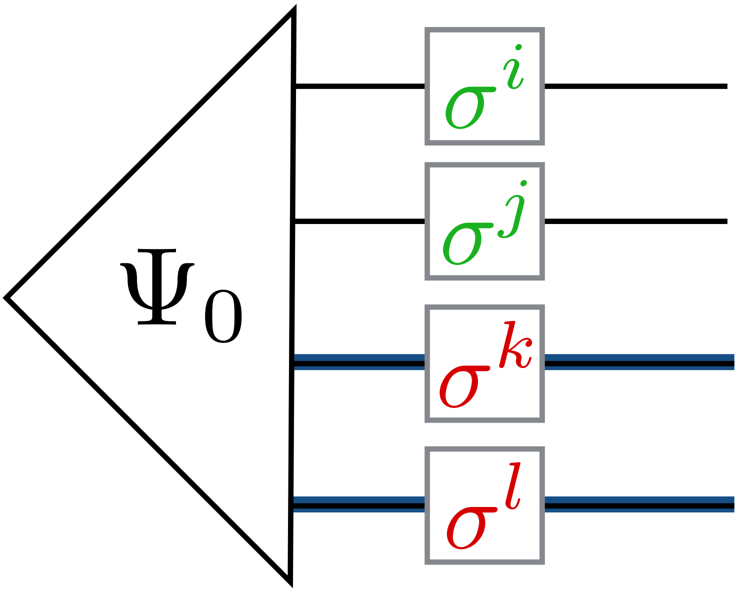

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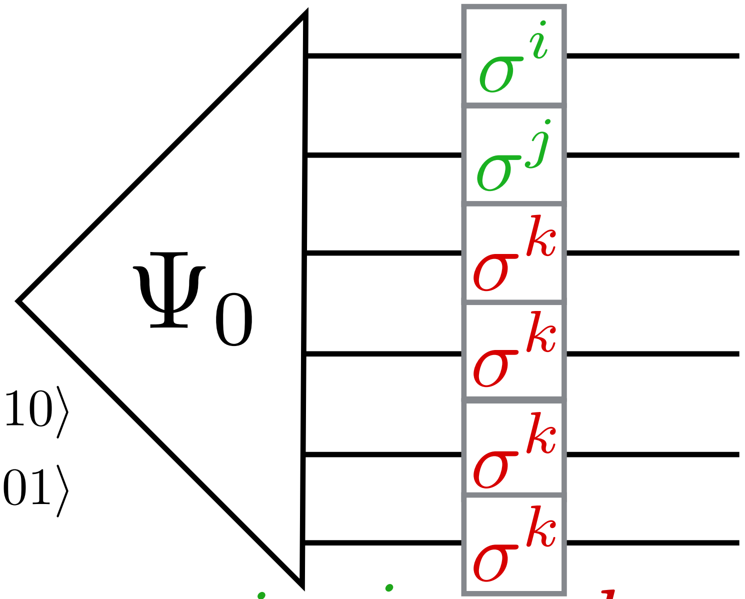

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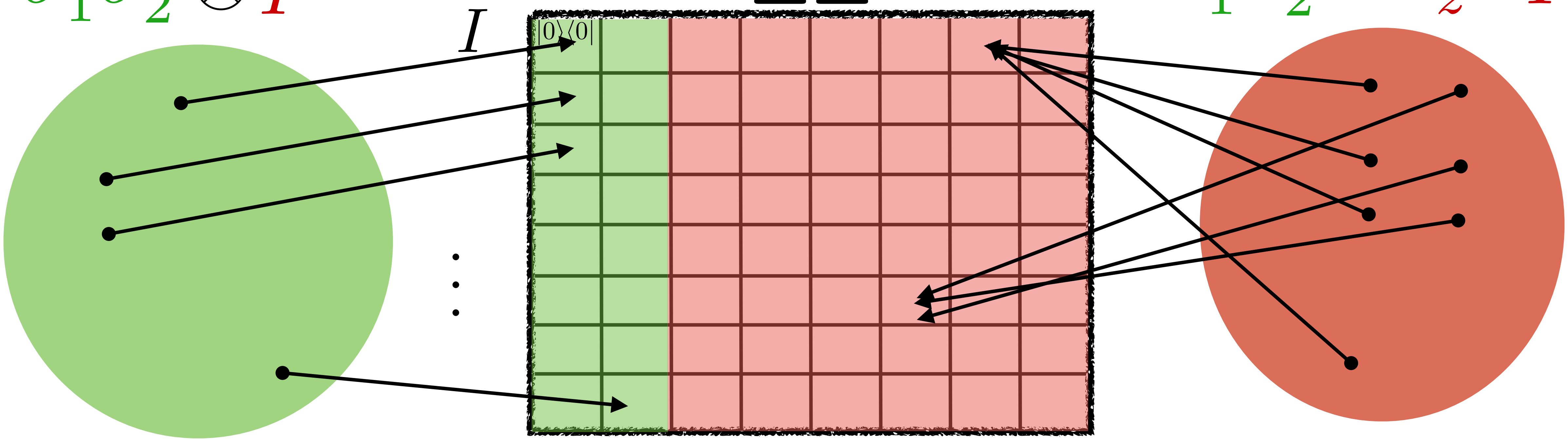

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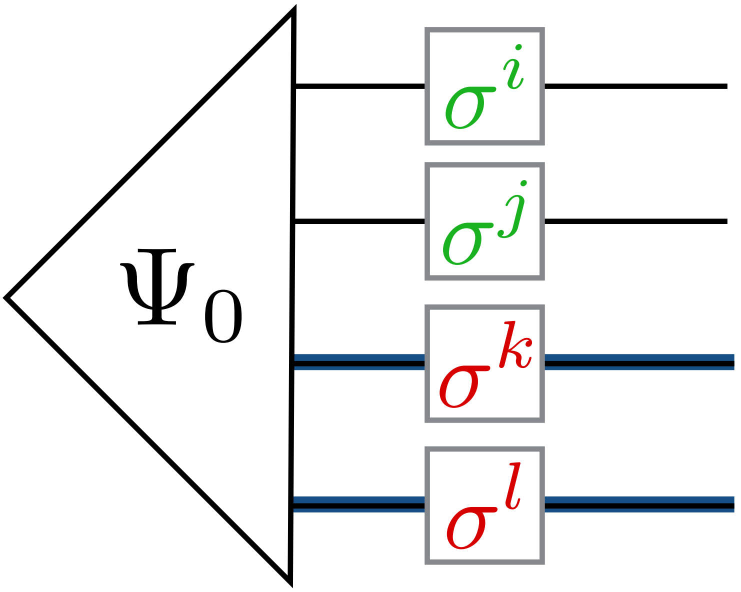

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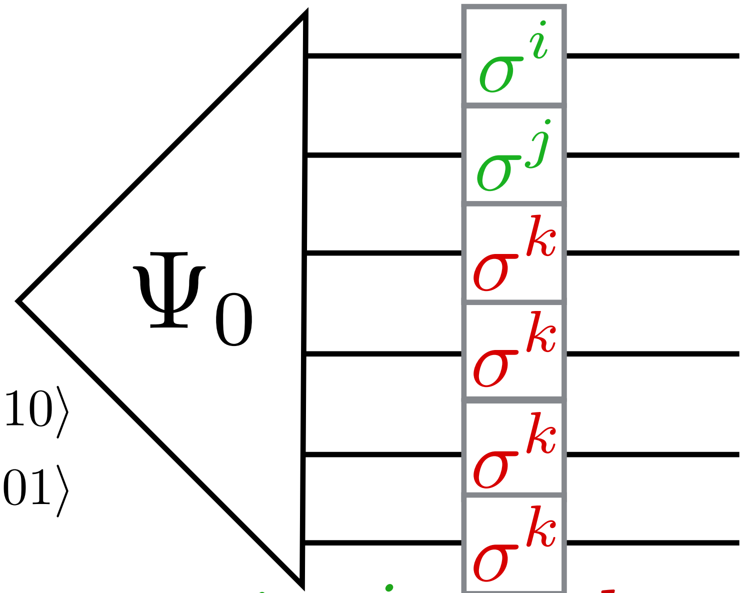
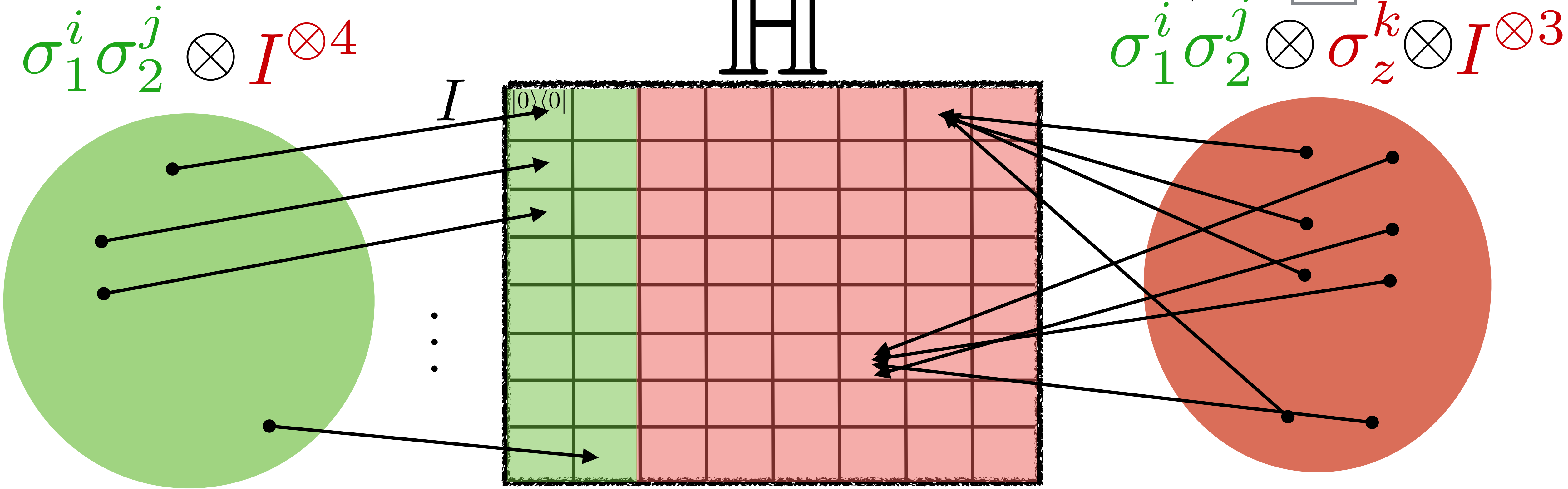
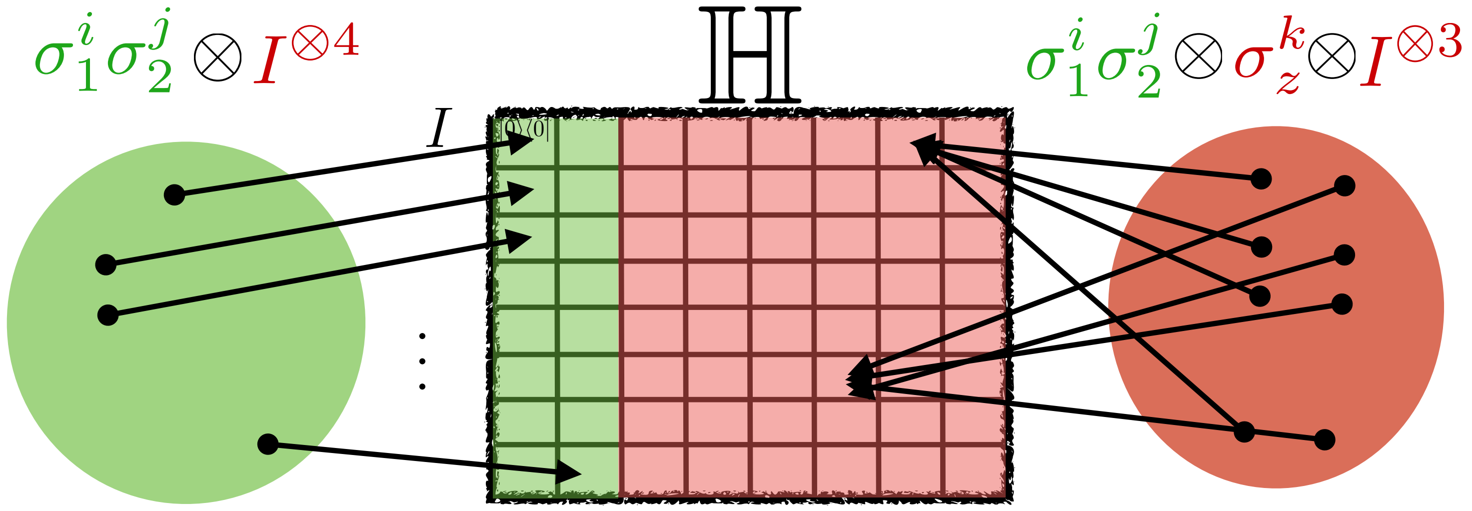




Selective Reconstruction

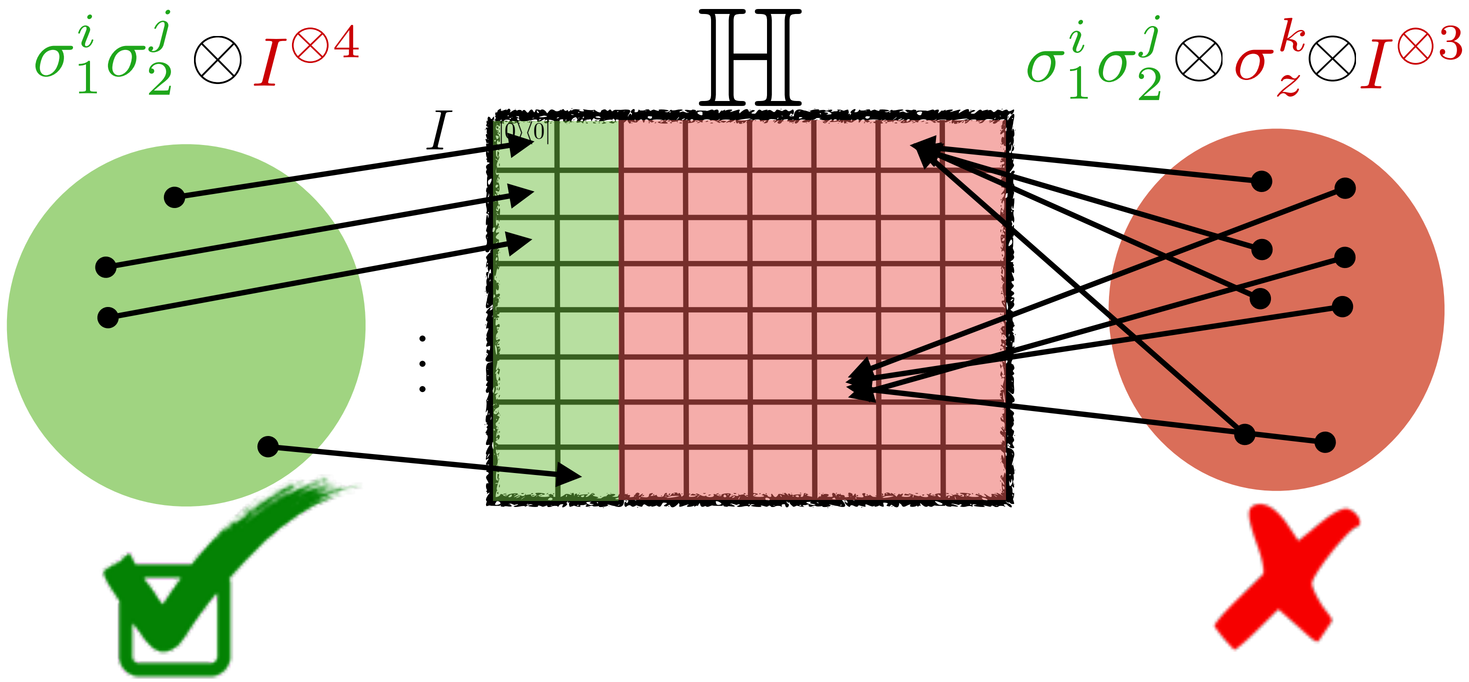


Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, ZIZIZI, IXIXIX, IZIIZZ \rangle .$$

i	E_i	e_i	i	E_i	e_i
0	$\mathbb{1}\mathbb{1}$	000000	8	XY	000111
1	$X\mathbb{1}$	000100	9	XZ	000110
2	$Y\mathbb{1}$	001100	10	YX	001101
3	$Z\mathbb{1}$	001000	11	YY	001111
4	$\mathbb{1}X$	000001	12	YZ	001110
5	$\mathbb{1}Y$	000011	13	ZX	001001
6	$\mathbb{1}Z$	000010	14	ZY	001011
7	XX	000101	15	ZZ	001010

Index **Operators** **Signature**



Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, ZIZIZI, IXIXIX, IZIIZZ\rangle.$$

i	E_i	e_i	i	E_i	e_i
0	$\mathbb{1}\mathbb{1}$	000000	8	XY	000111
1	$X\mathbb{1}$	000100	9	XZ	000110
2	$Y\mathbb{1}$	001100	10	YX	001101
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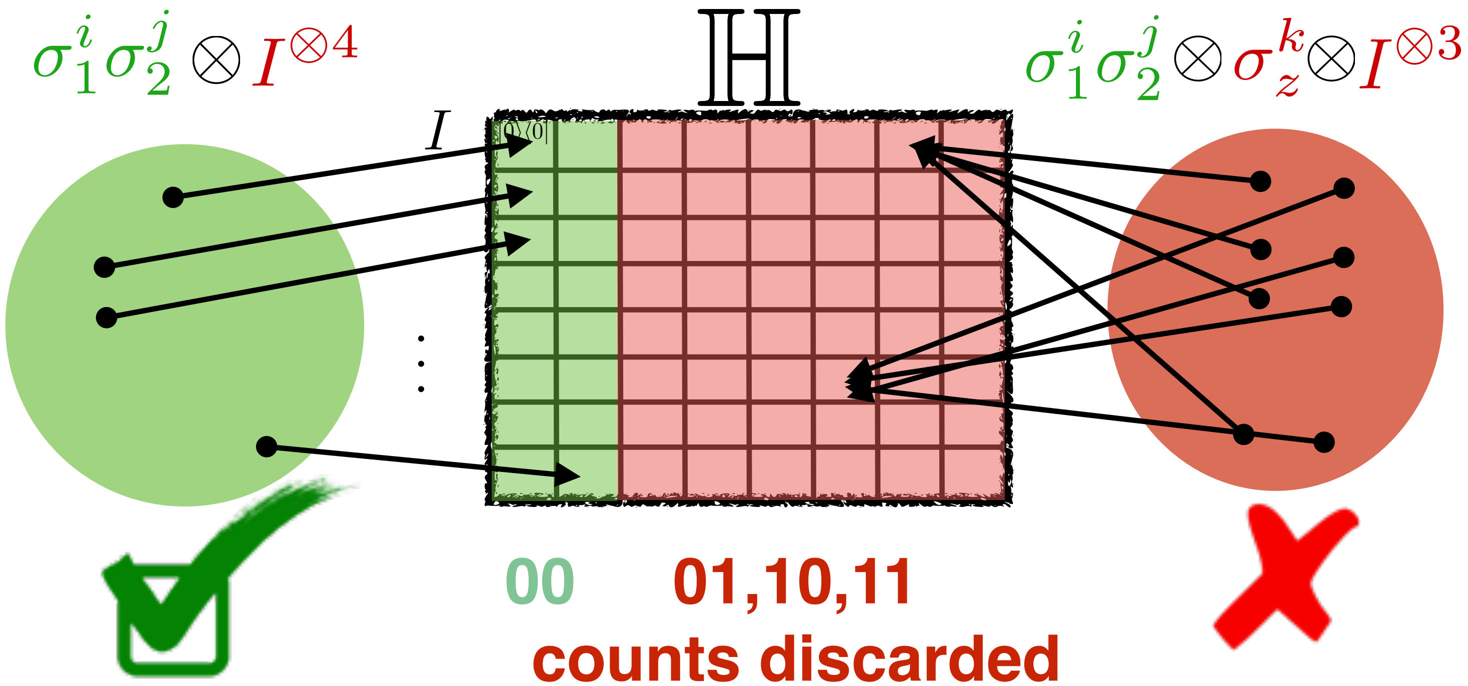

00 **01,10,11**

counts discarded

=> increased SNR

Selective Reconstruction

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1	$X\mathbb{1}$	000100	9	XZ	000110
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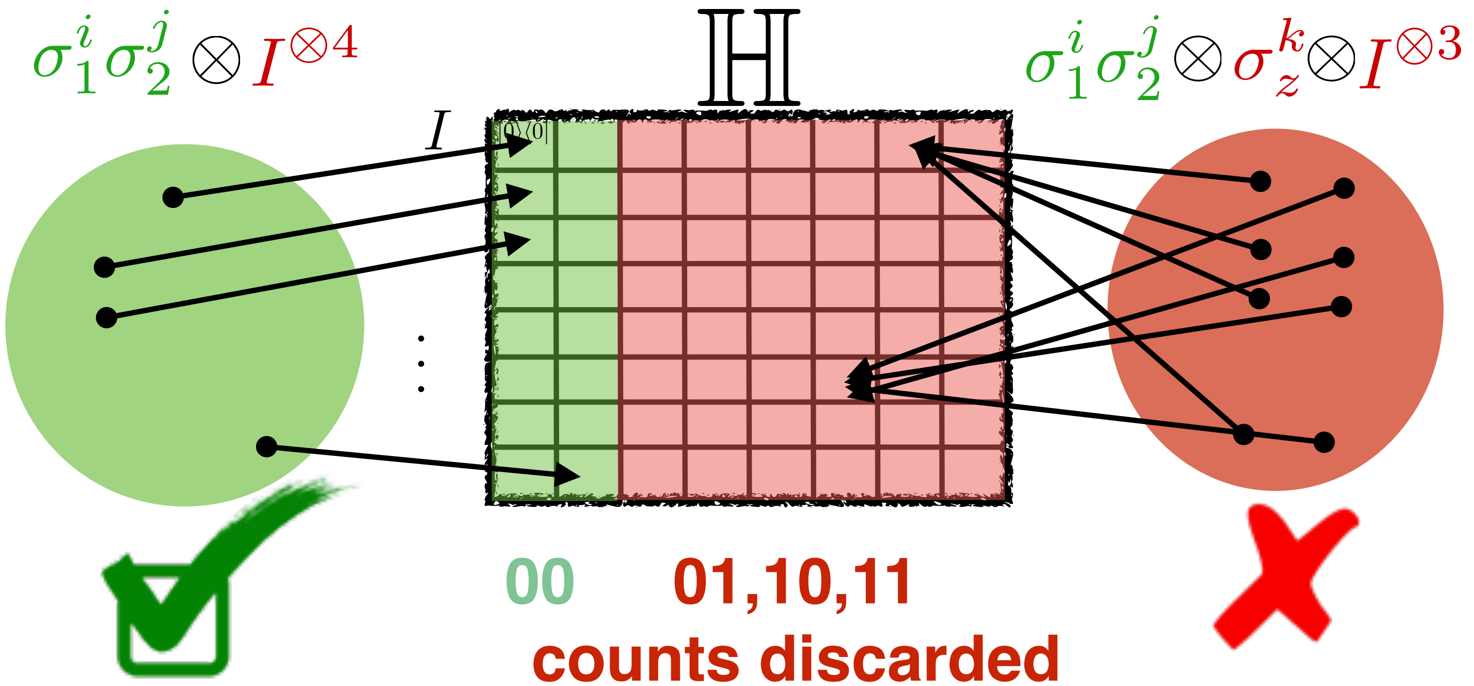

00 **01,10,11**

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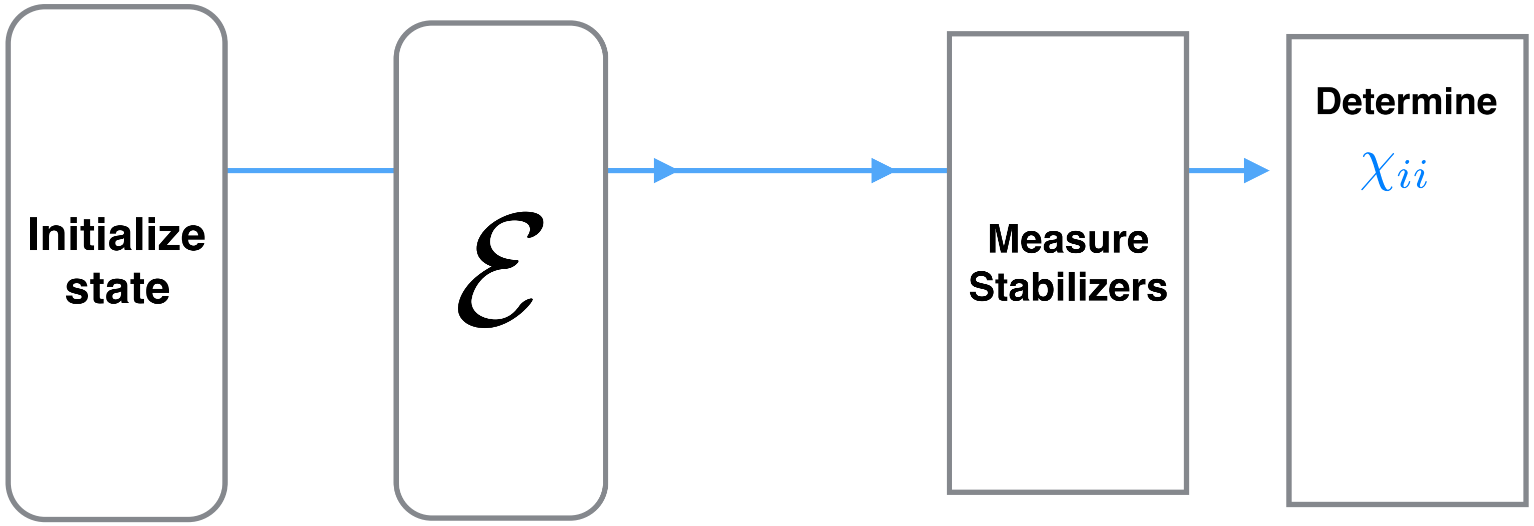
$$p_i = \chi_{ii}$$



Initialize state

$\mathcal{E}$

Measure Stabilizers

Determine χ_{ii}

Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

i	E_i	e_i	i	E_i	e_i
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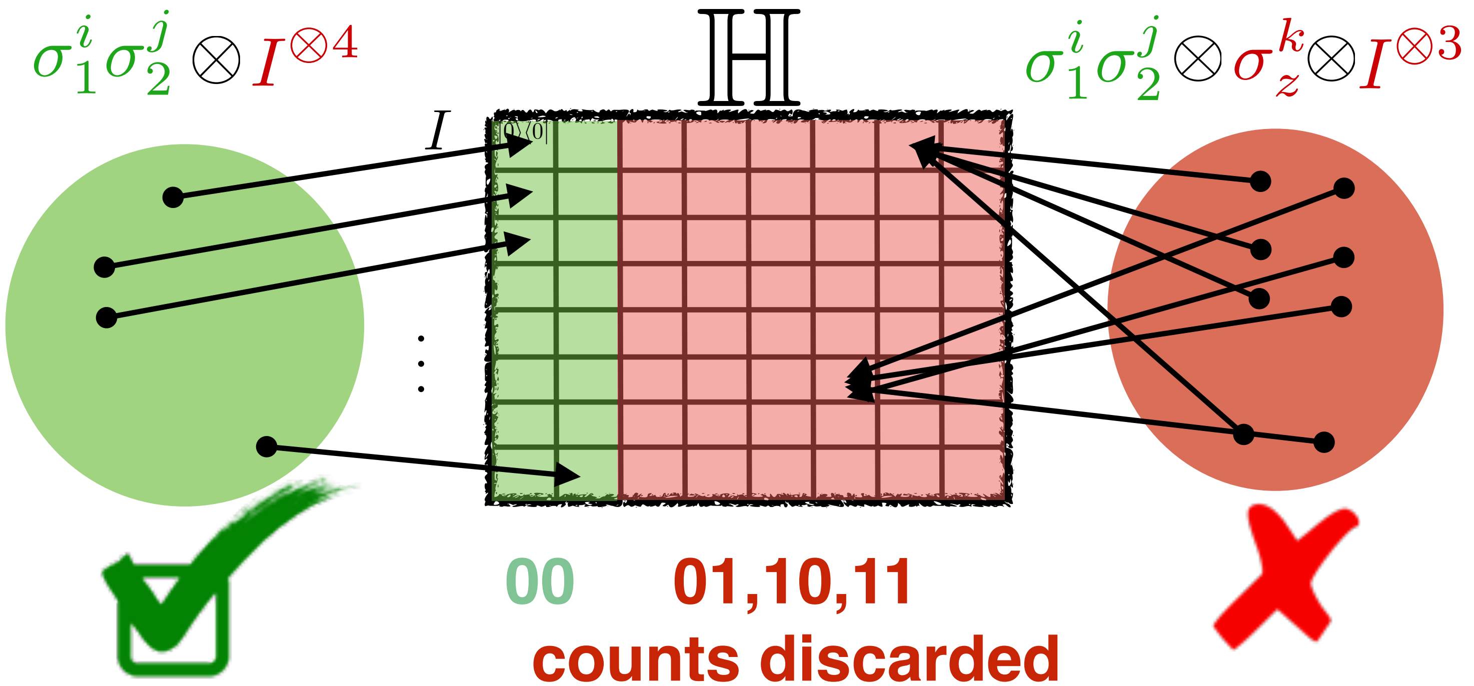

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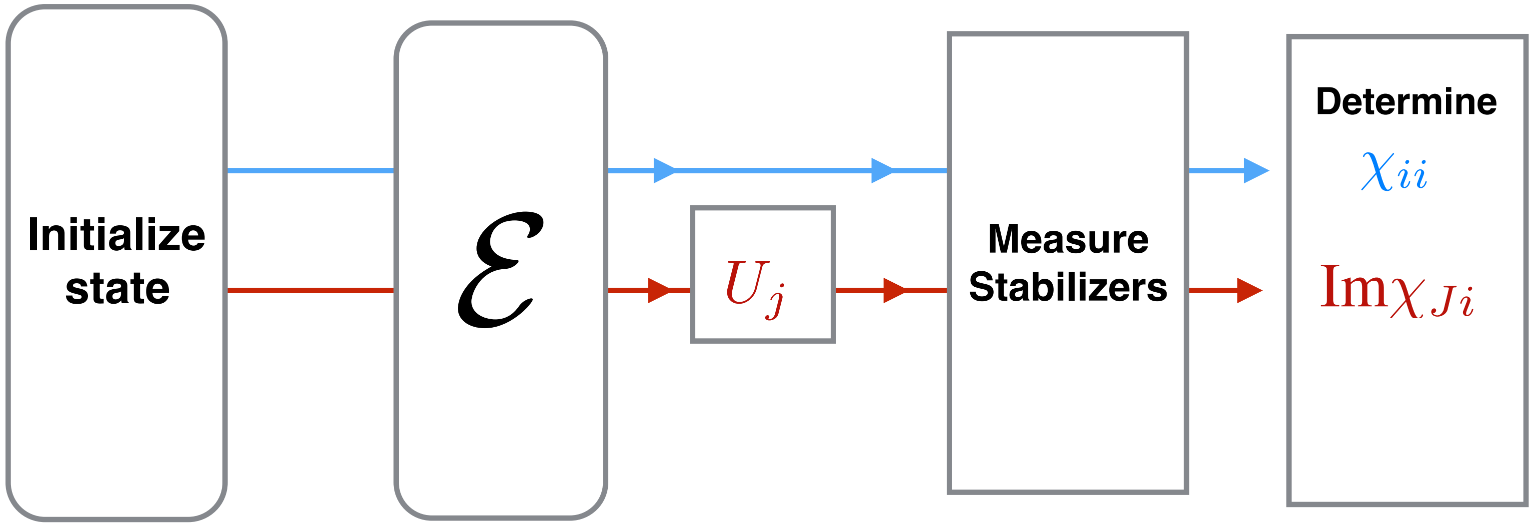

$$p_i = \chi_{ii}$$

$$p_i(U_j) = \frac{\chi_{ii} + \chi_{JJ}}{2} - \mathrm{Im}\left(\phi_J \chi_{Ji}\right)$$

Selective Reconstruction

$$\mathcal{S}_1 = \langle IIXXXX, IIZZZZ, XIXXII, ZIZIZI, IXIXIX, IZIIZZ \rangle.$$

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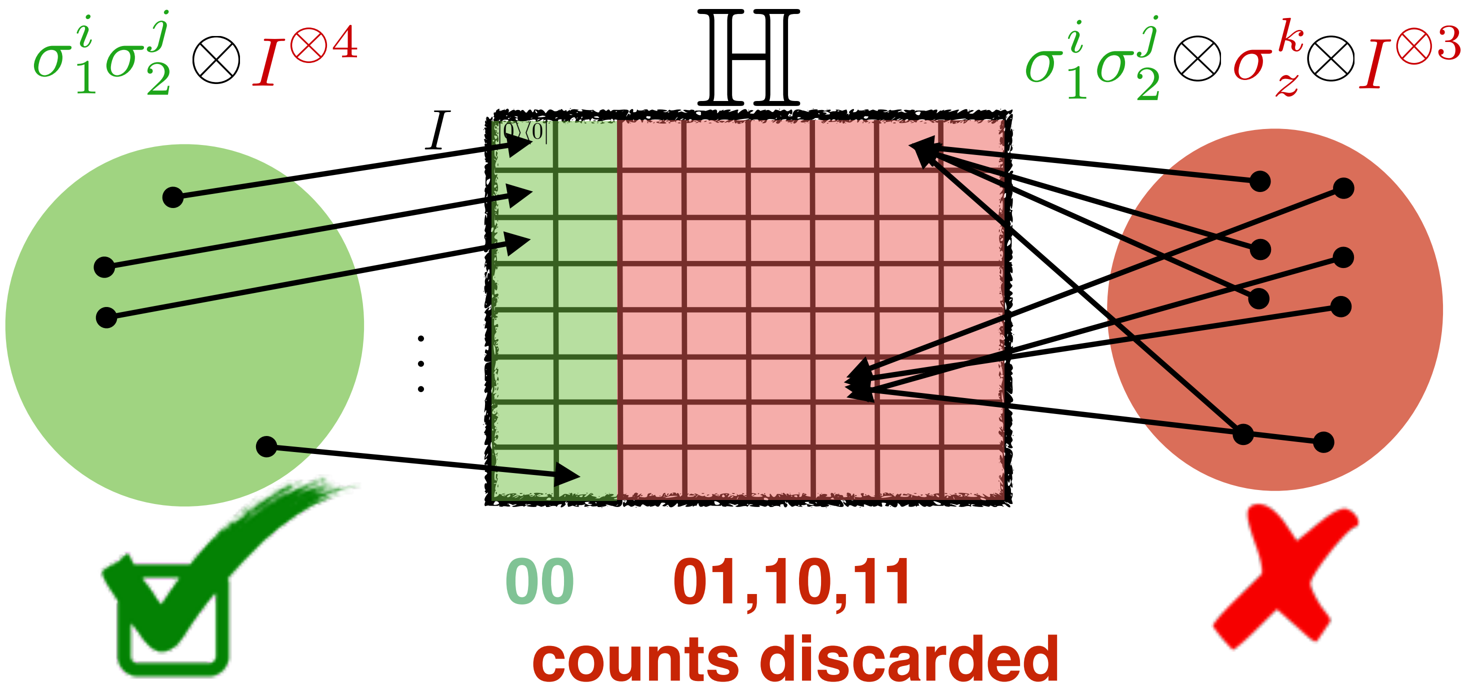

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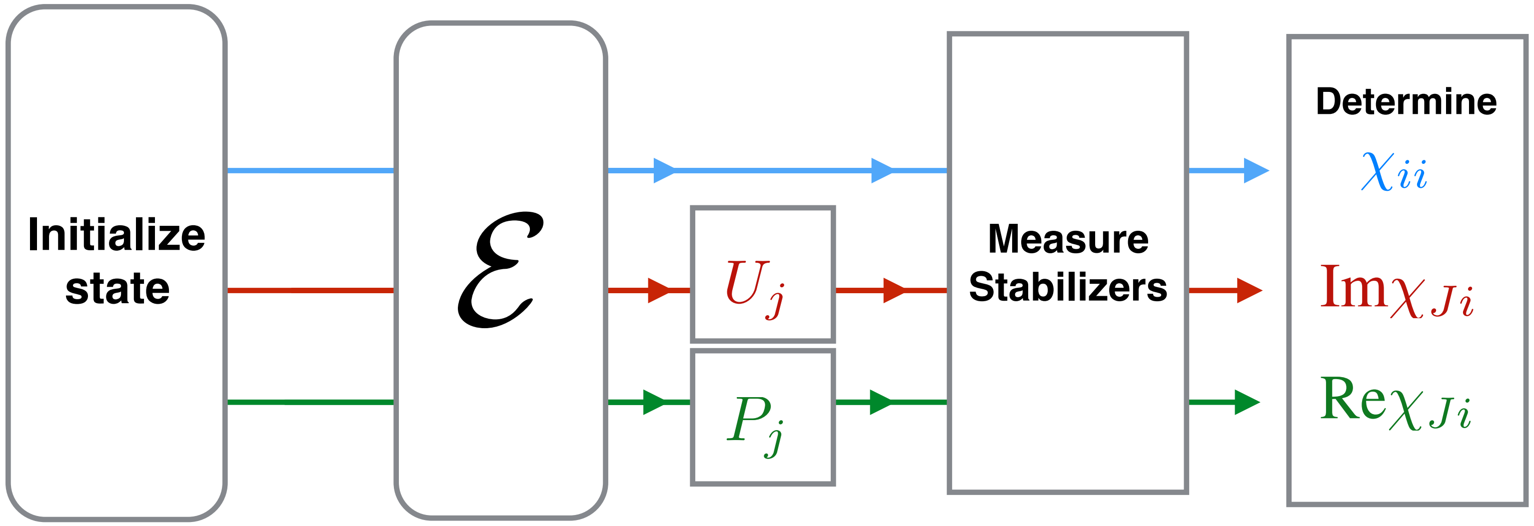

Initialize state

$\mathcal{E}$

U_j

P_j

Measure Stabilizers

Determine
χ_{ii}
$\mathrm{Im}\chi_{Ji}$
$\mathrm{Re}\chi_{Ji}$

$$p_i = \chi_{ii}$$

$$p_i(U_j) = \frac{\chi_{ii} + \chi_{JJ}}{2} - \mathrm{Im}\left(\phi_J \chi_{Ji}\right)$$

$$p_i(P_{j\pm}) = \frac{\chi_{ii} + \chi_{JJ}}{2} \pm \mathrm{Re}\left(\phi_J \chi_{Ji}\right)$$

Selective Reconstruction

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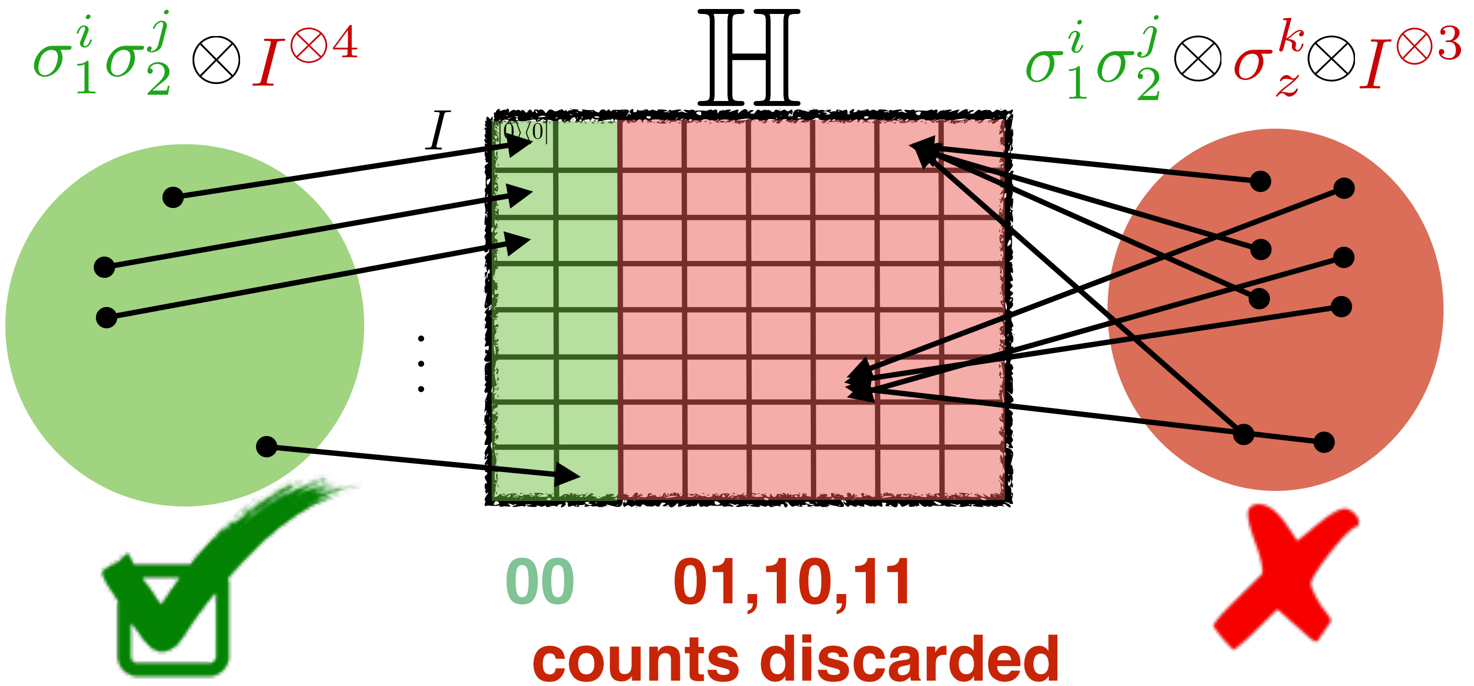

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counts discarded

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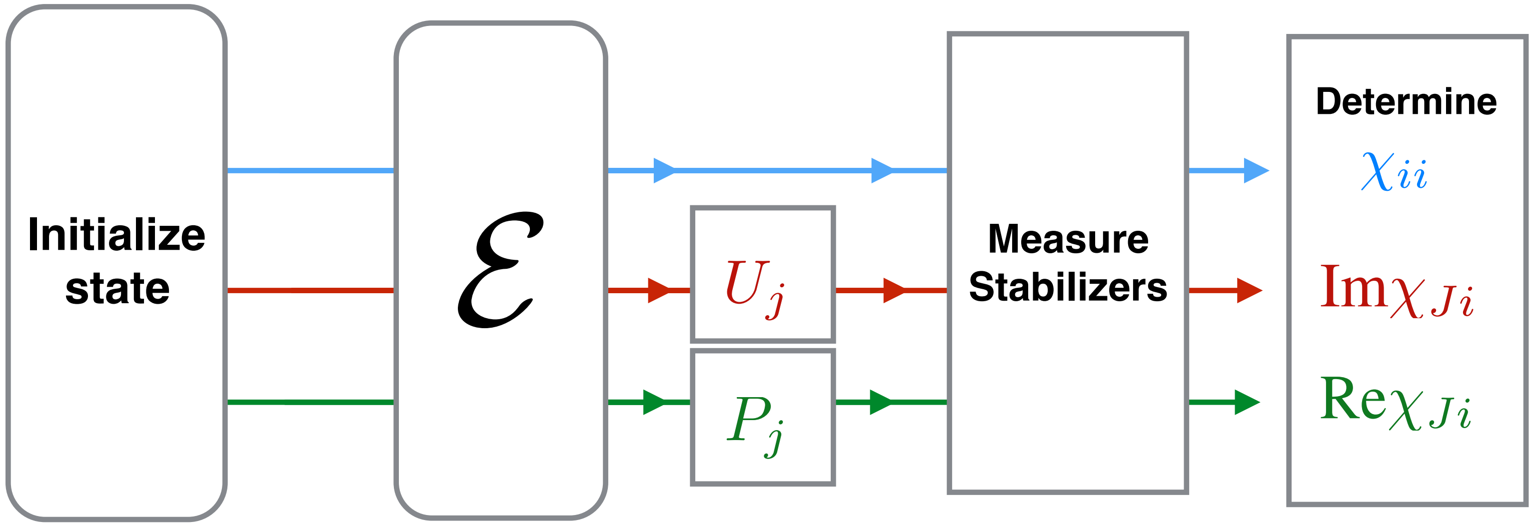

Initialize state

$\mathcal{E}$

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P_j

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$\mathrm{Re}\chi_{Ji}$

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$$\mathcal{E}(\rho) = \sum_{m,n} \chi_{mn} F_i \rho F_i^\dagger$$

Ex: Single Qubit Process

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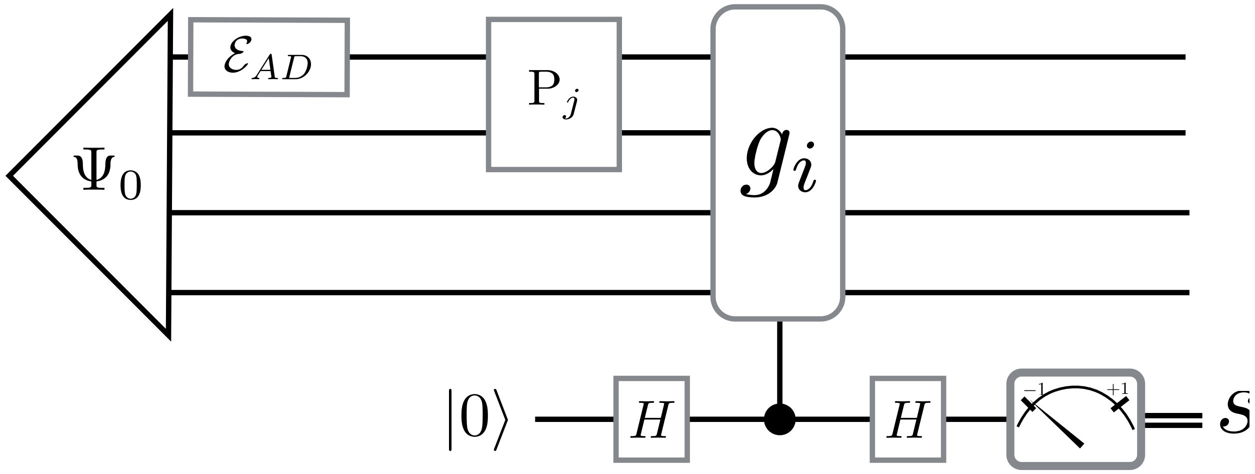

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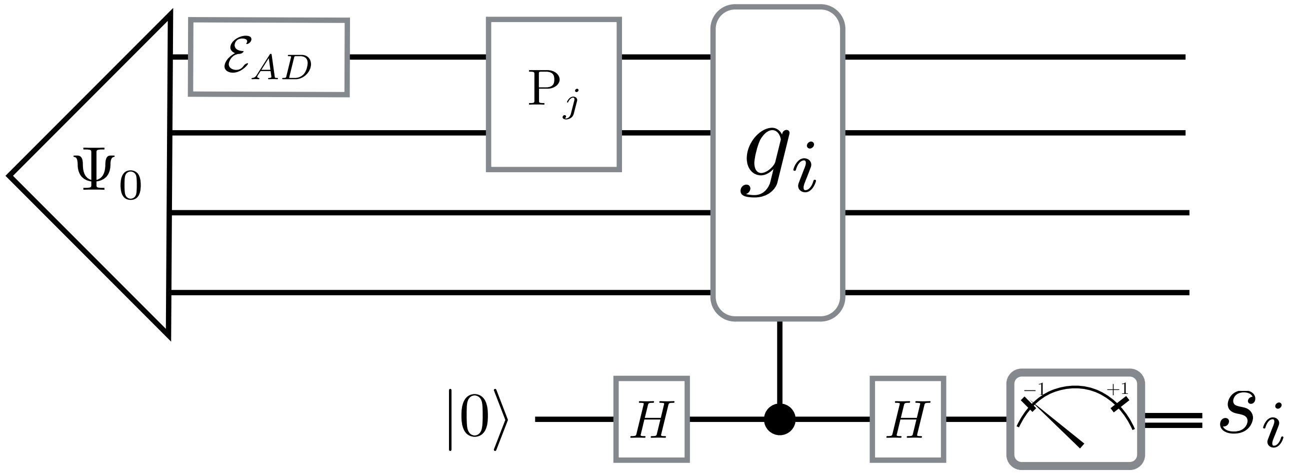
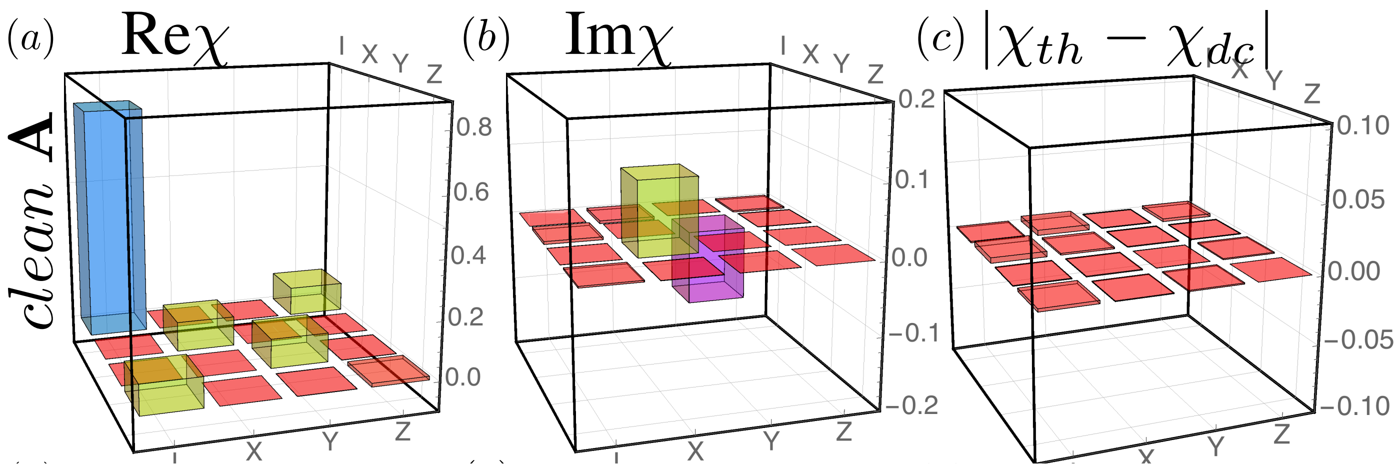
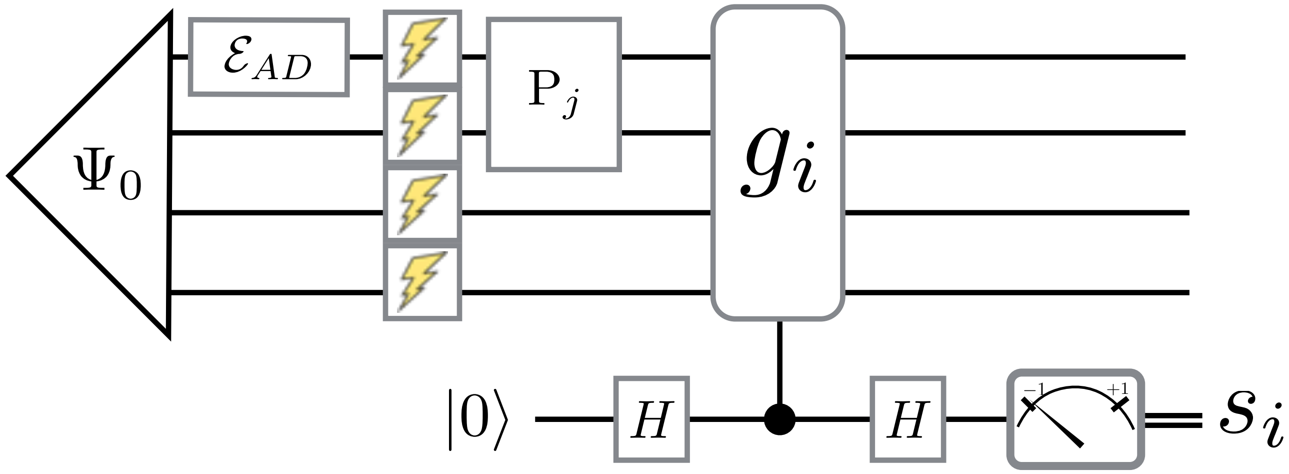
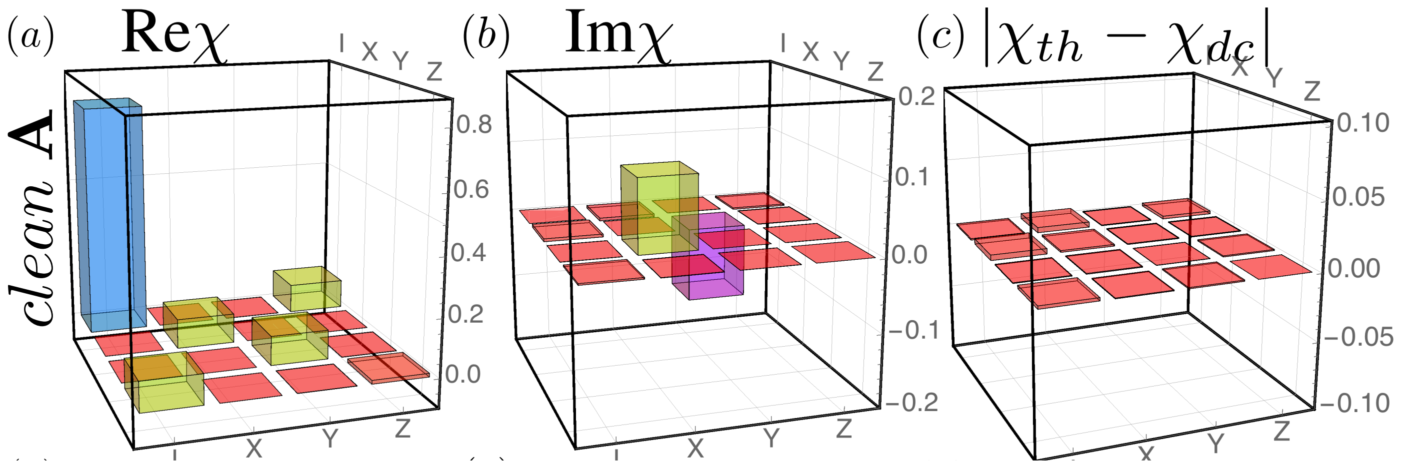




Ex: Single Qubit Process

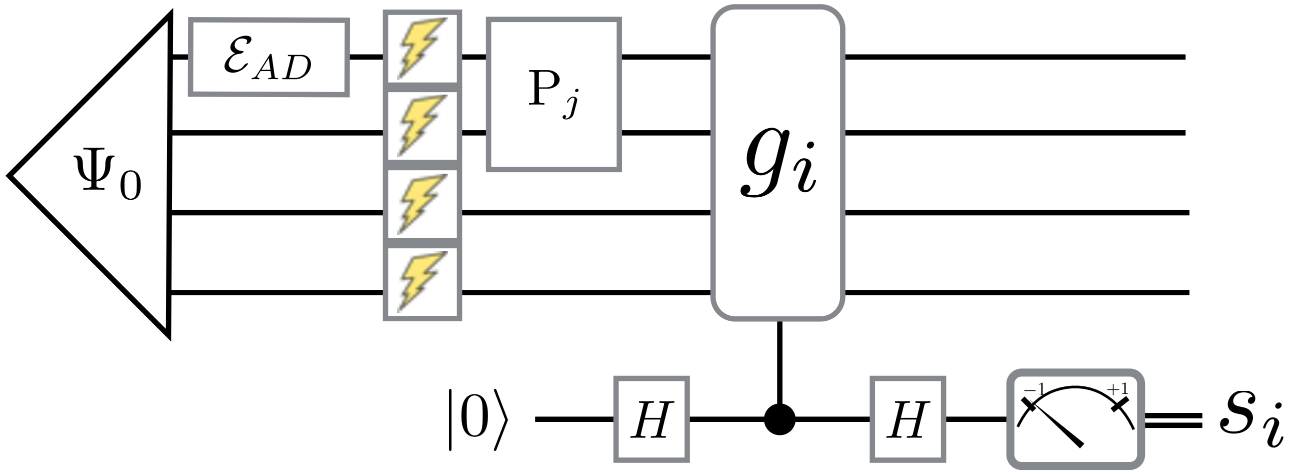
Depolarizing Noise $p_{DP} = 0.1$

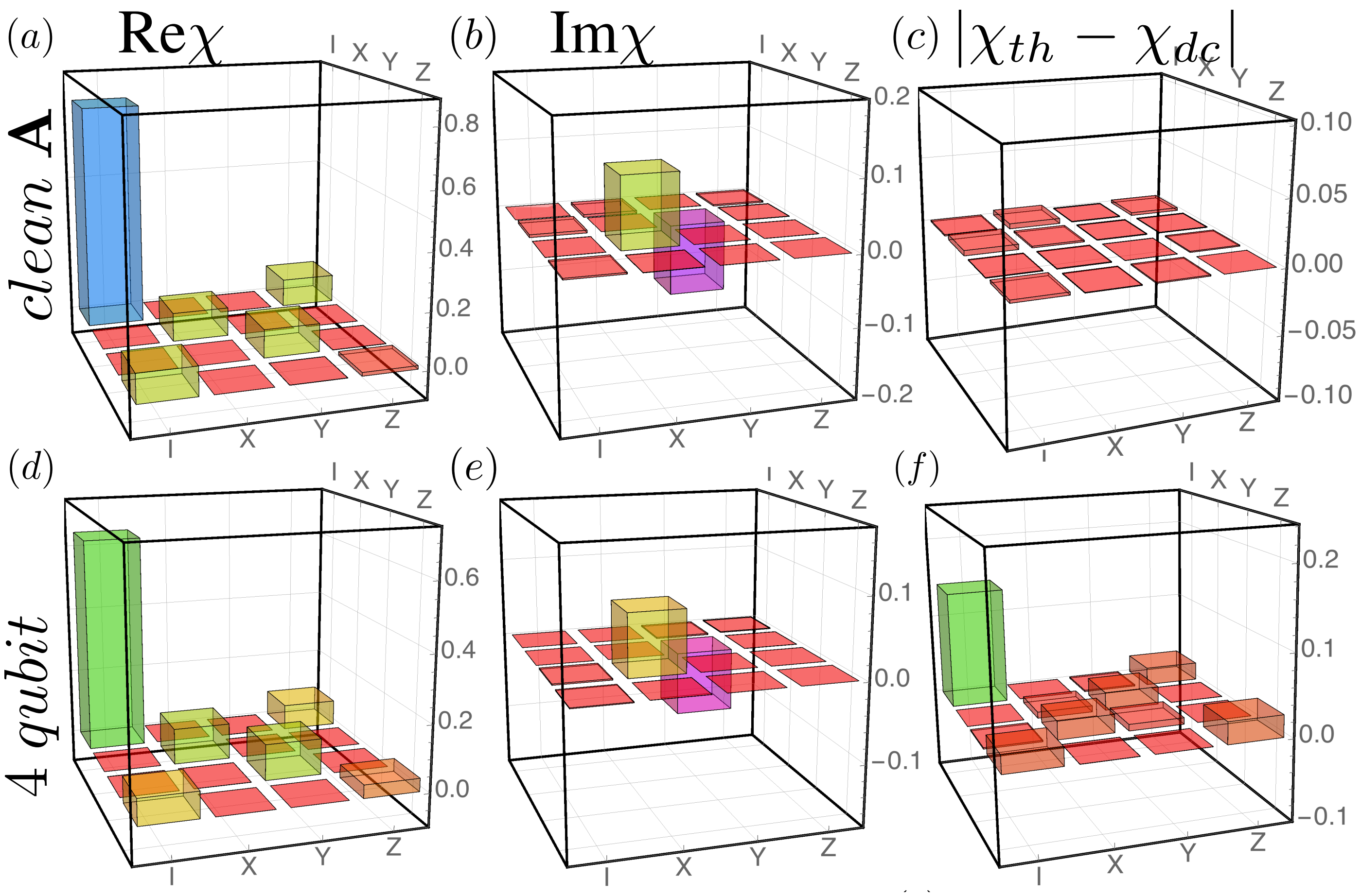




Ex: Single Qubit Process

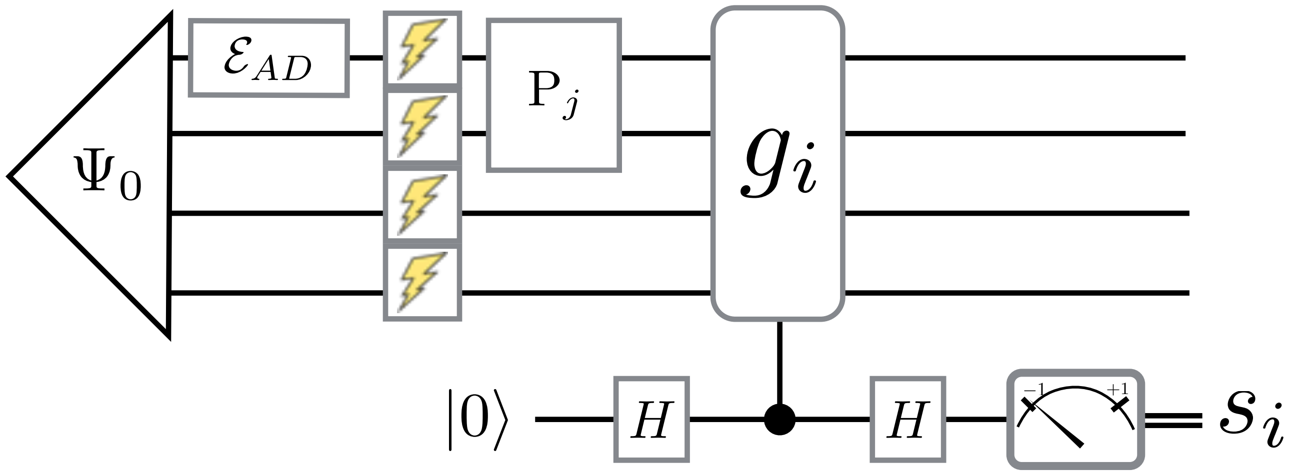
Depolarizing Noise $p_{DP} = 0.1$



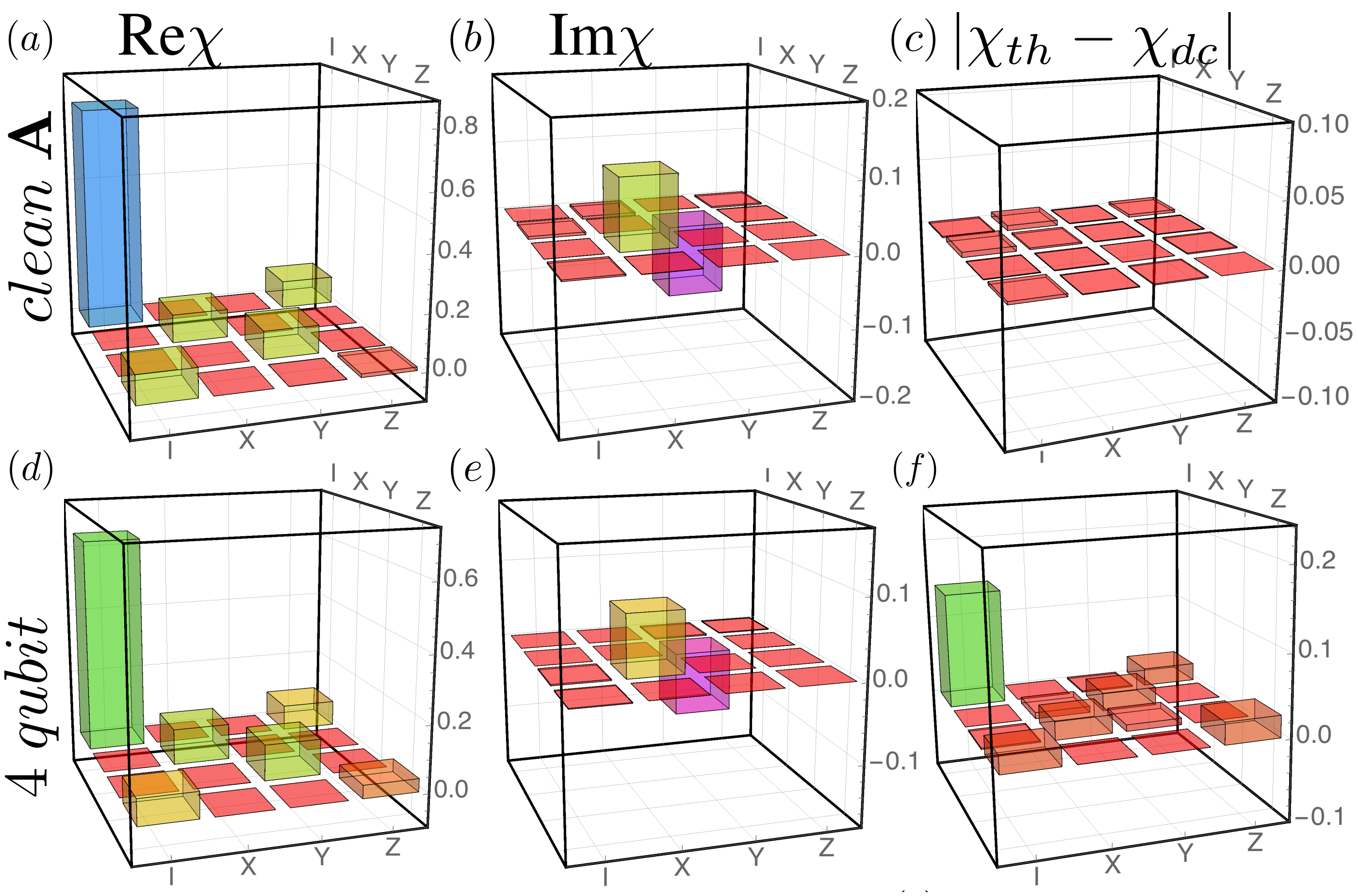


Ex: Single Qubit Process

Depolarizing Noise $p_{DP} = 0.1$

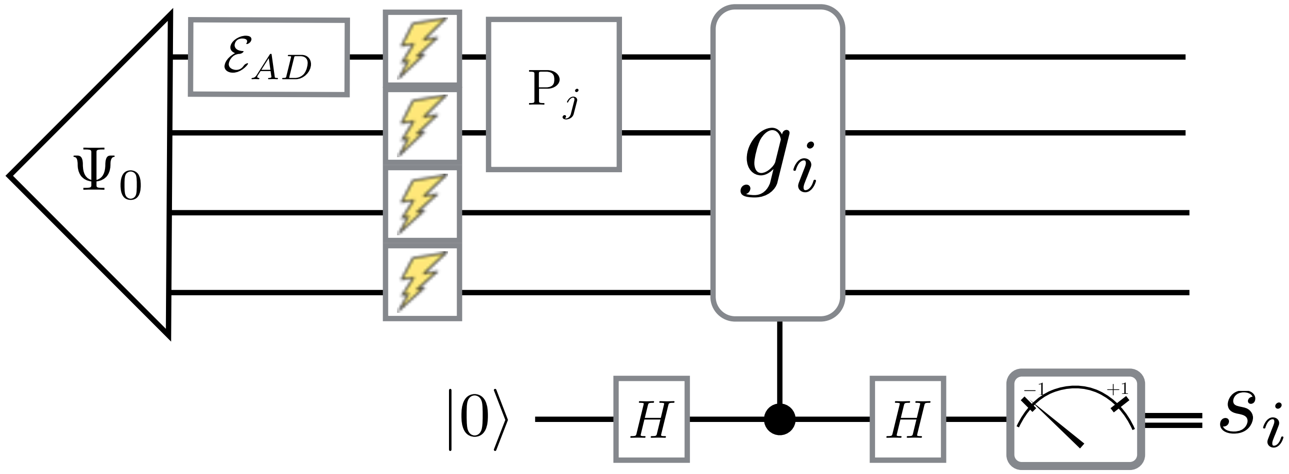


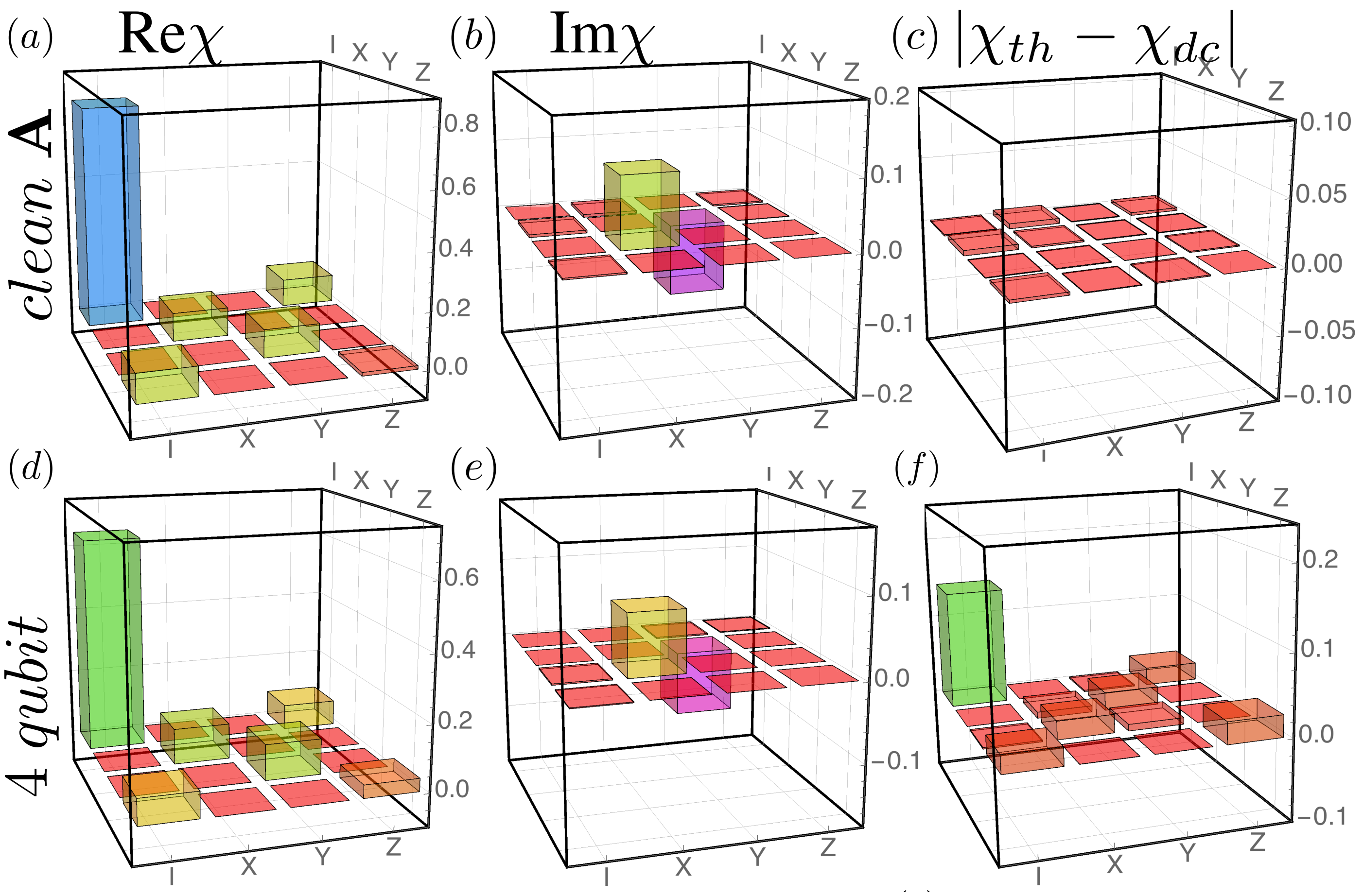
$$F\left[\mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho)\right] = .9165$$



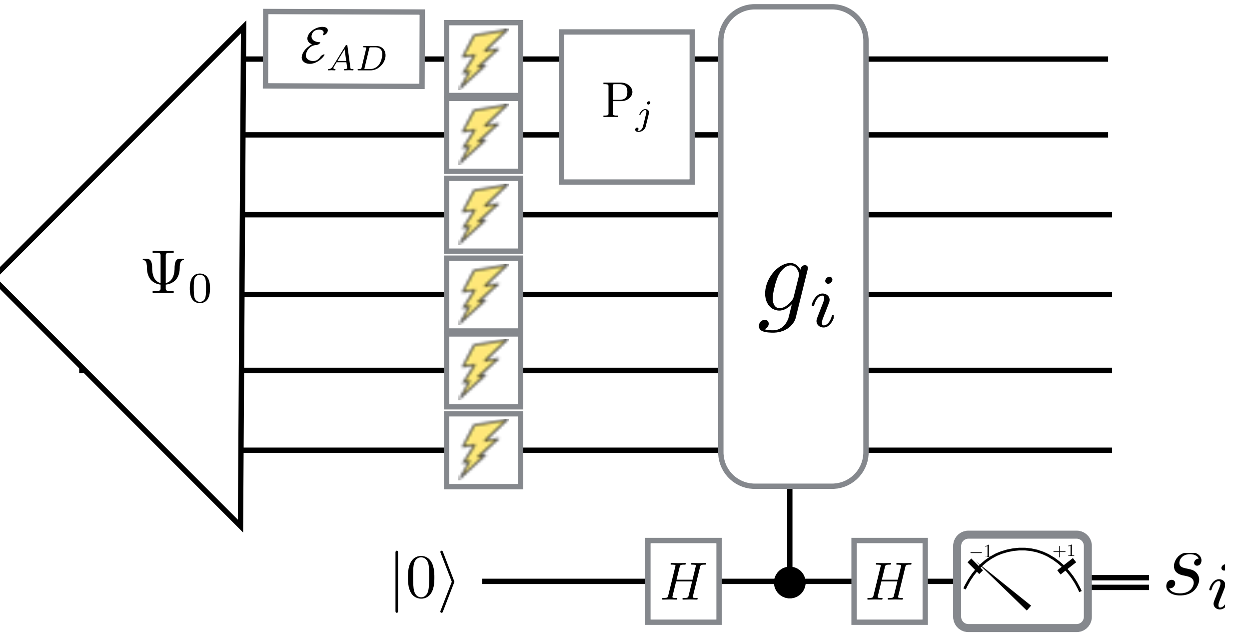
Ex: Single Qubit Process

Depolarizing Noise $p_{DP} = 0.1$



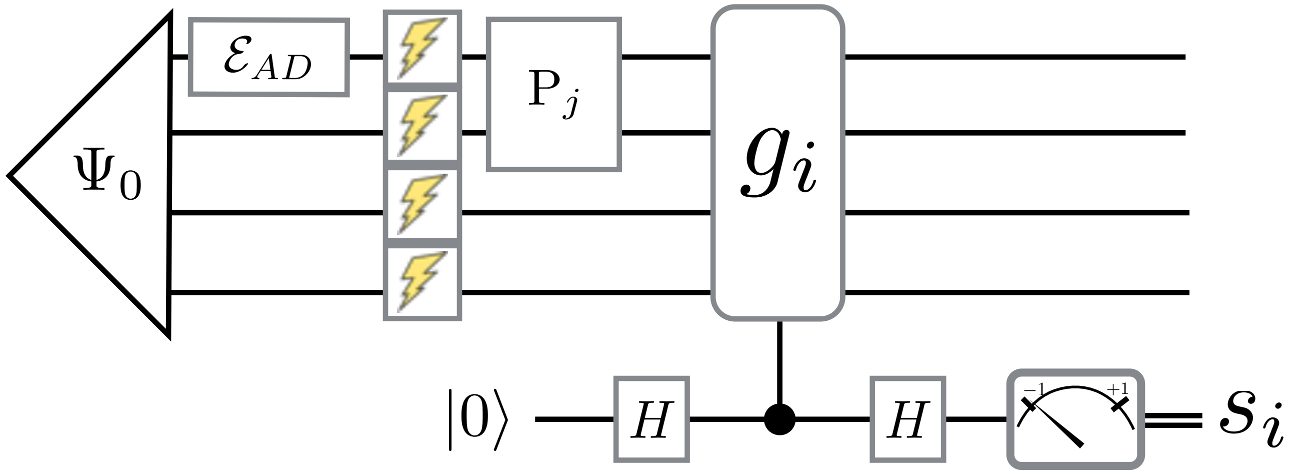


$$F\left[\mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho)\right] = .9165$$

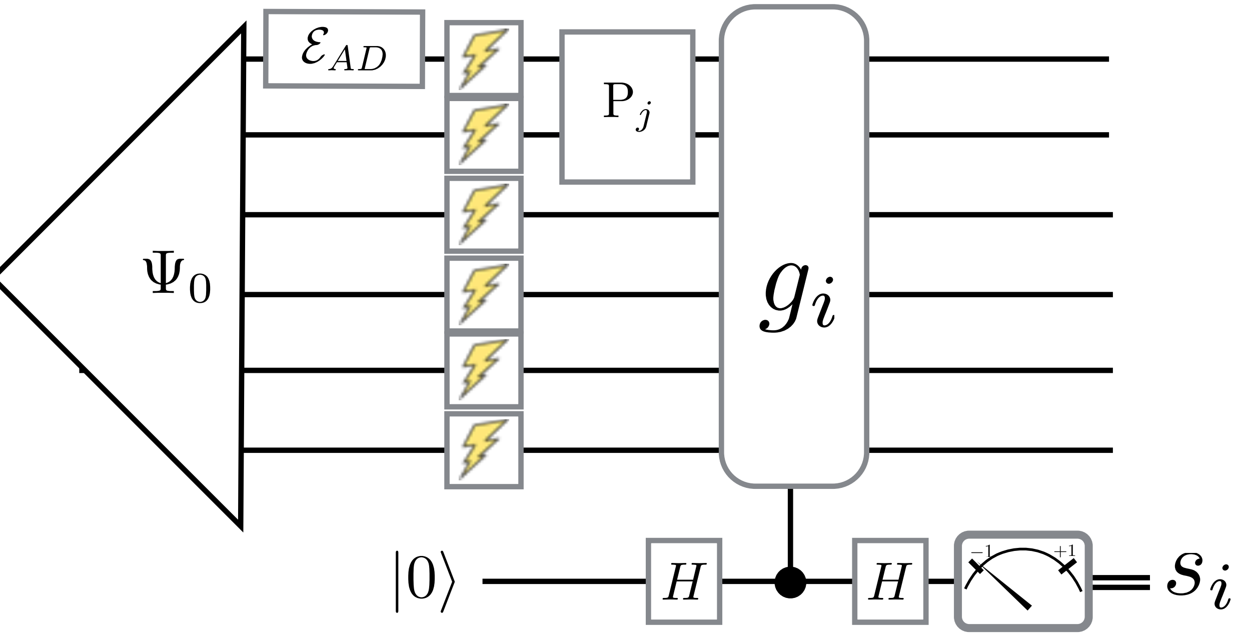


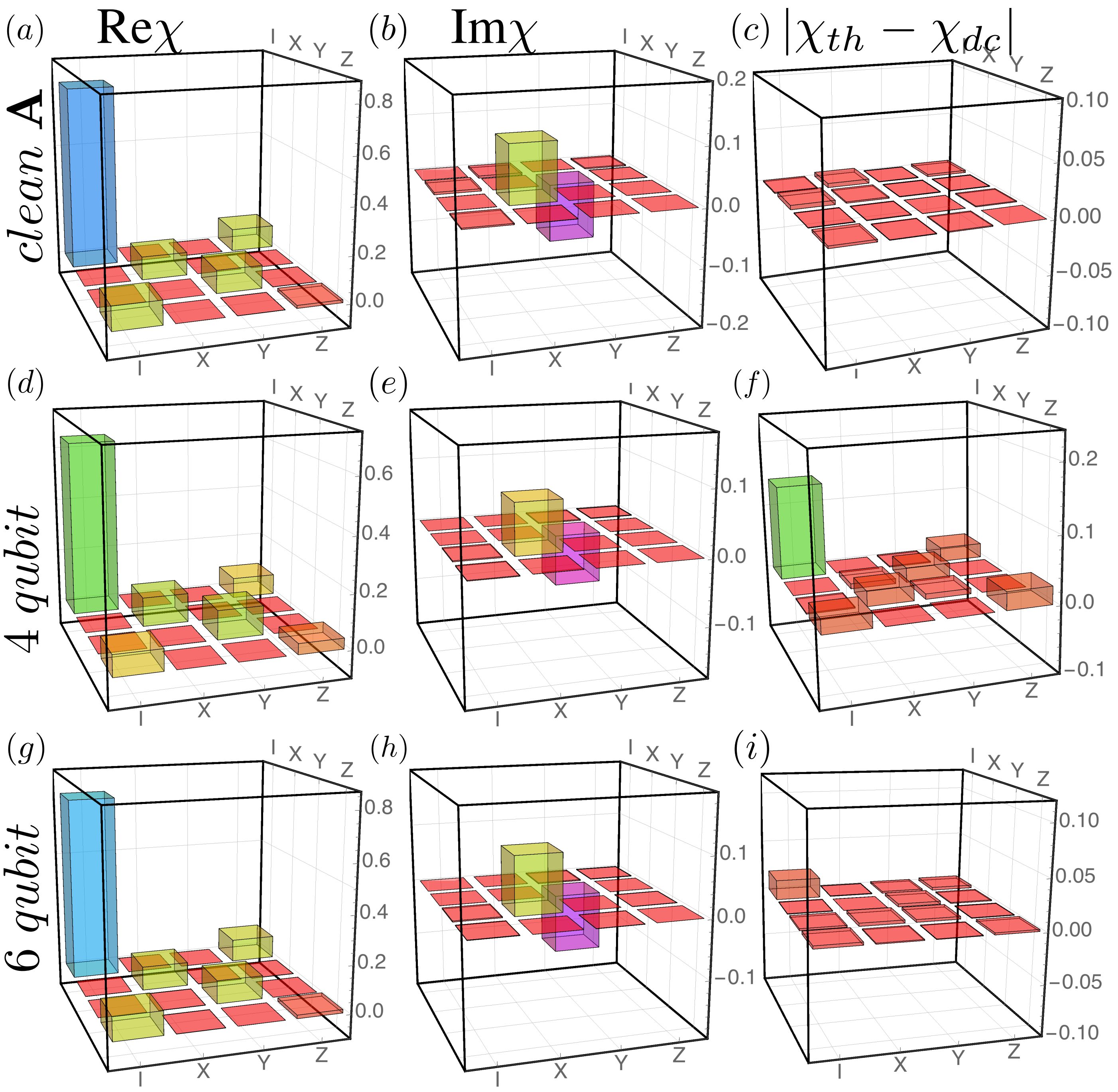
Ex: Single Qubit Process

Depolarizing Noise $p_{DP} = 0.1$



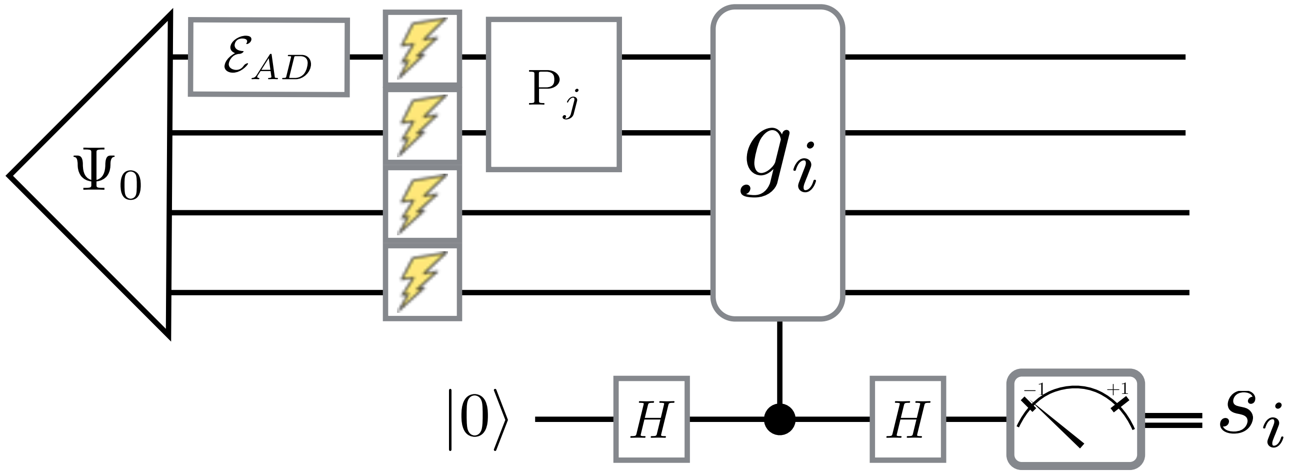
$$F\left[\mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho)\right] = .9165$$



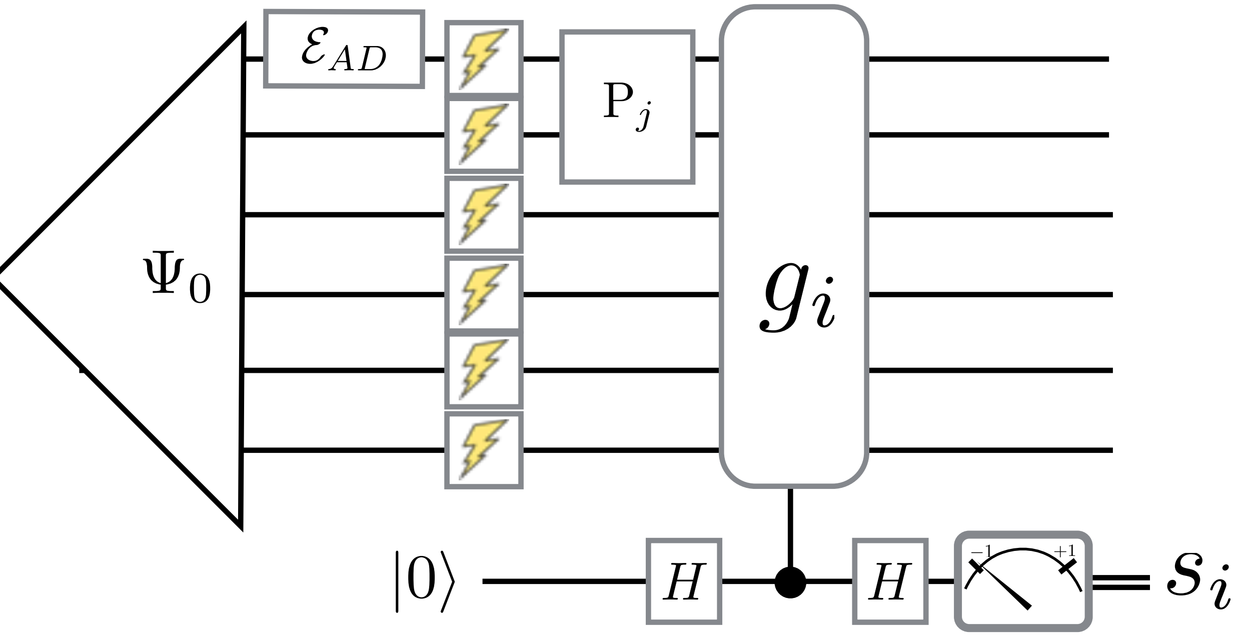


Ex: Single Qubit Process

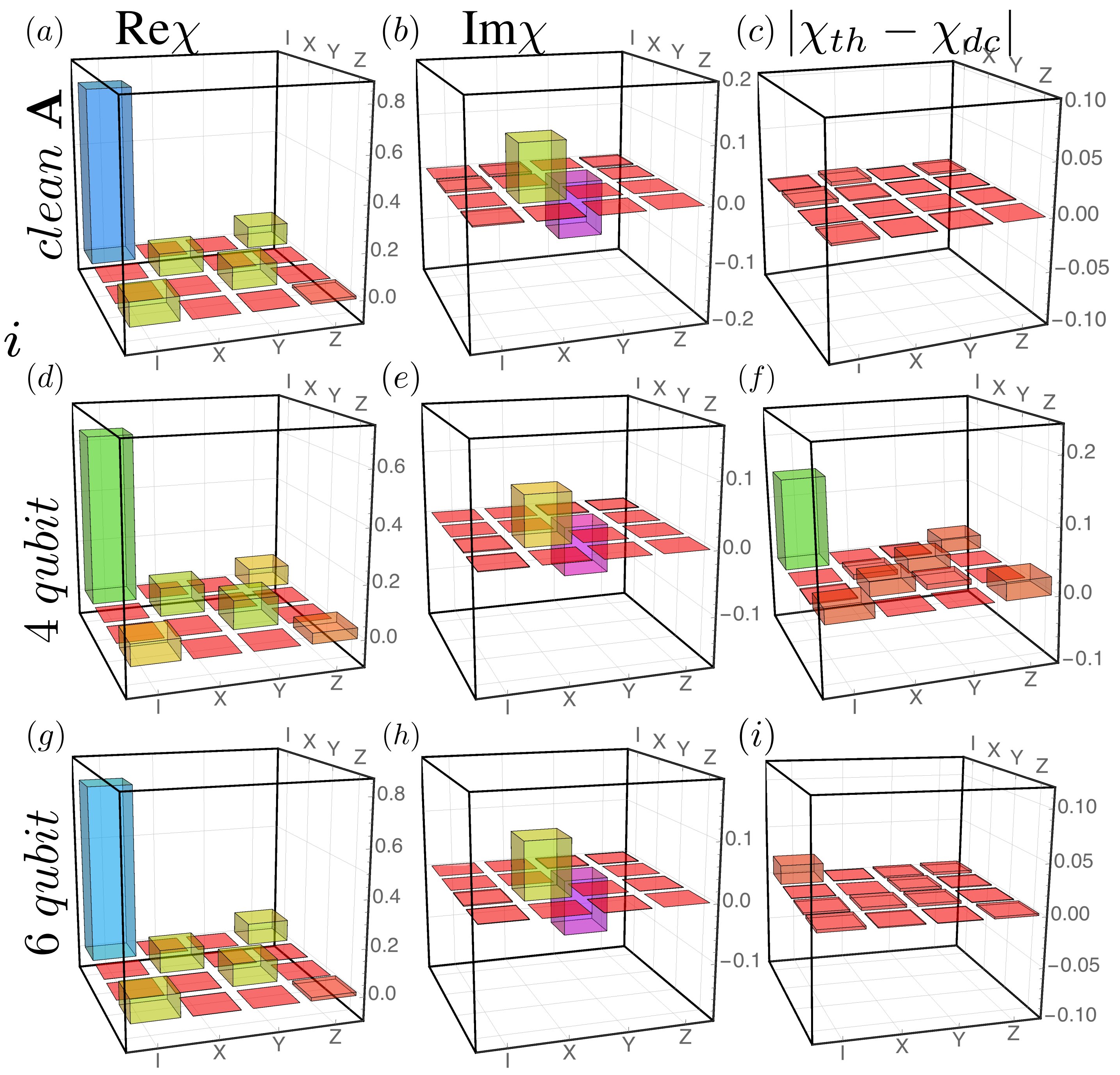
Depolarizing Noise $p_{DP} = 0.1$



$$F\left[\mathcal{E}^{AD}(\rho), \mathcal{E}^{[[4,0,2]]}(\rho)\right] = .9165$$



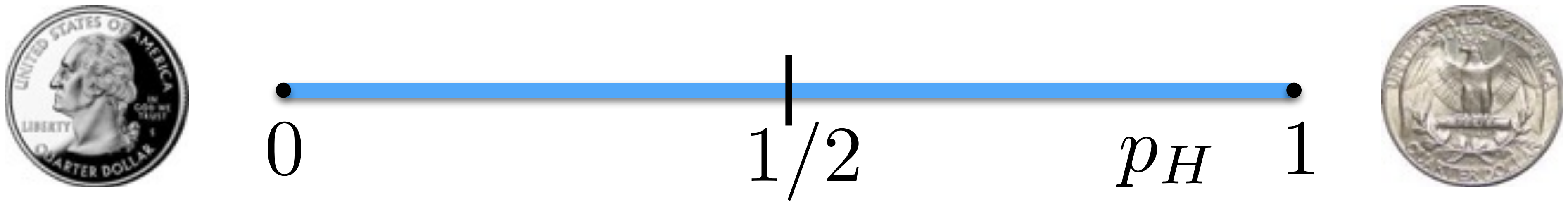
$$F\left[\mathcal{E}^{AD}(\rho), \mathcal{E}^{[[6,0,2]]}(\rho)\right] = .9884$$



Classical Model Selection

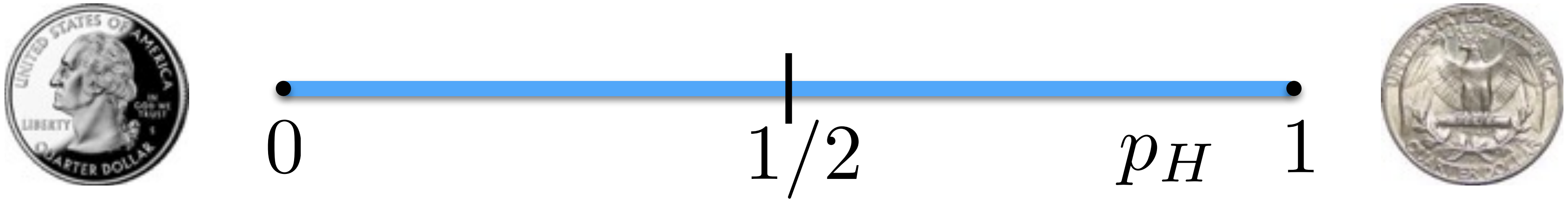
Classical Model Selection

Q: Is coin fair?



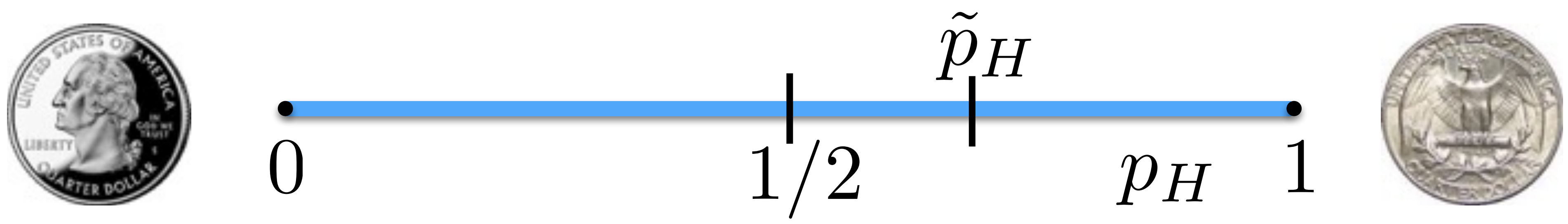
Classical Model Selection

Q: Is coin fair?



Classical Model Selection

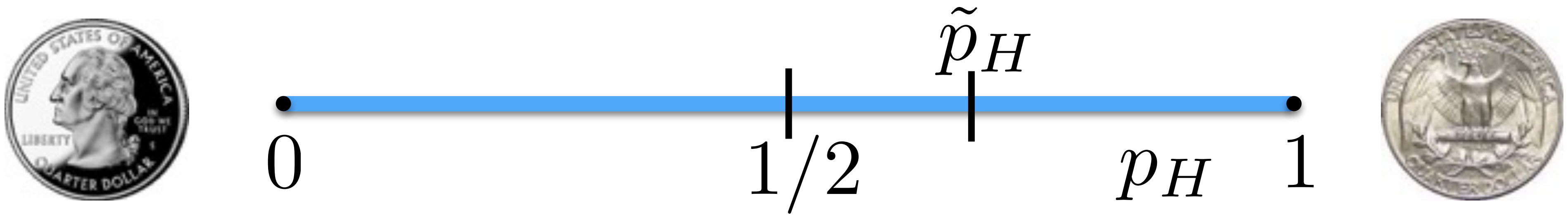
Q: Is coin fair?



$$\tilde{p}_H = \frac{N_H}{N_{tot}}$$

Classical Model Selection

Q: Is coin fair?



$X =$

$$\tilde{p}_H = \frac{N_H}{N_{tot}}$$

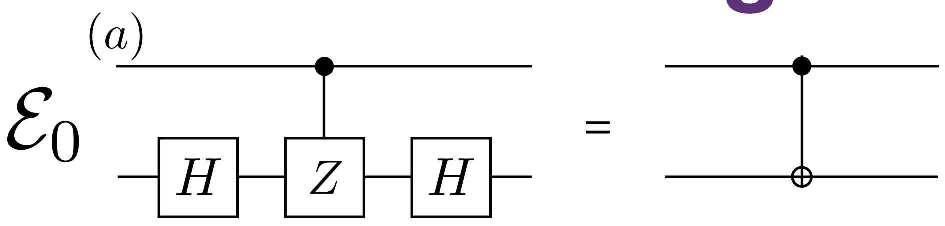
Choice:

$$\text{Log}\left(\frac{\mathcal{L}_{alt}}{\mathcal{L}_{null}} = \frac{Prob(X|p_H = 1/2)}{Prob(X|p_H = \tilde{p}_H)}\right) \begin{cases} < D^* & \text{null} \\ \geq D^* & \text{alternative} \end{cases}$$

Quantum Channel Discrimination

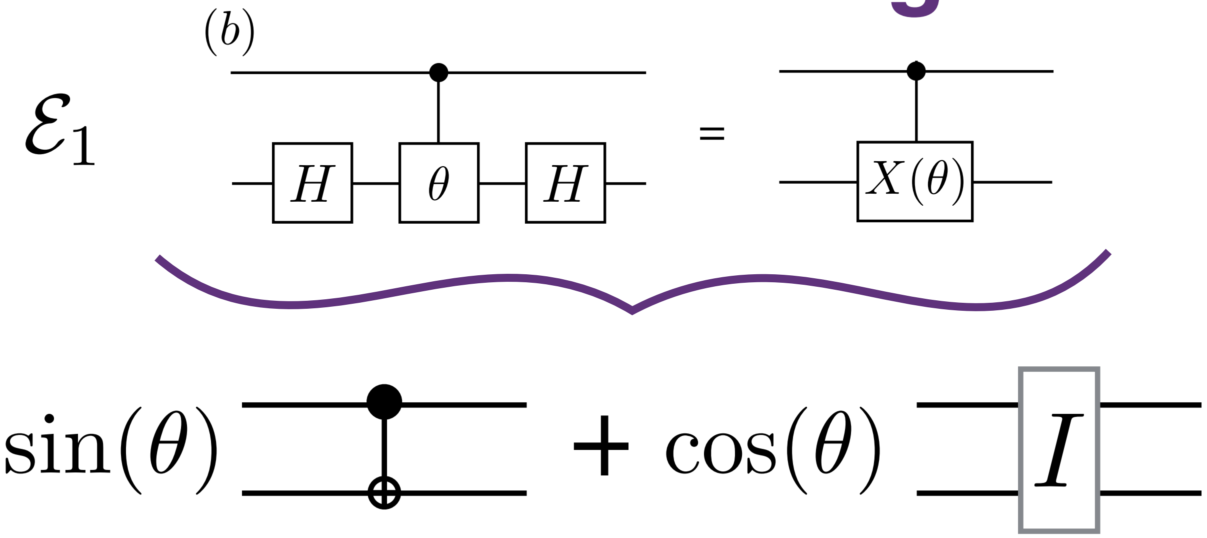
Quantum Channel Discrimination

Perfect Entangler:



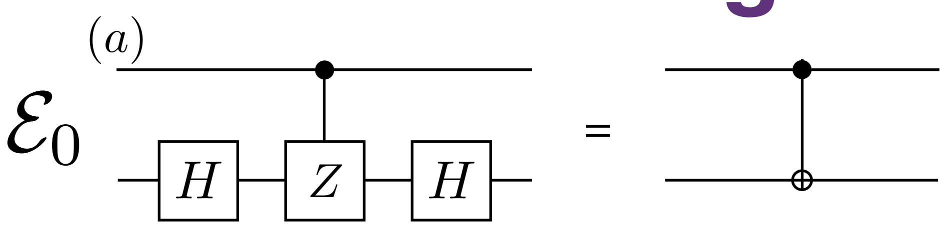
vs.

Partial Entangler:



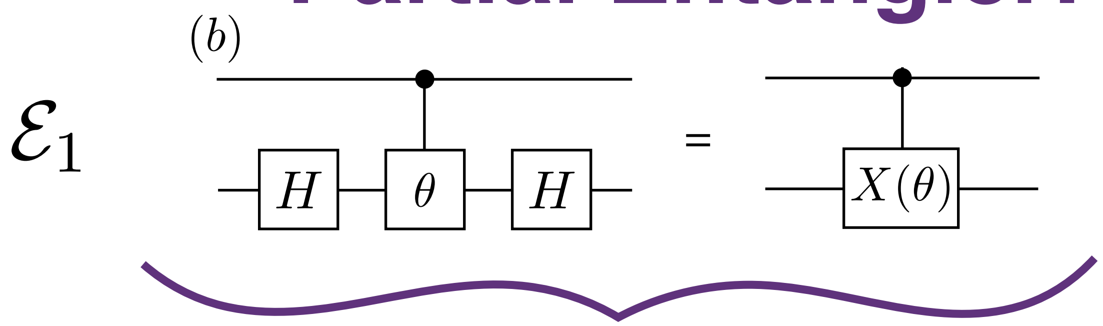
Quantum Channel Discrimination

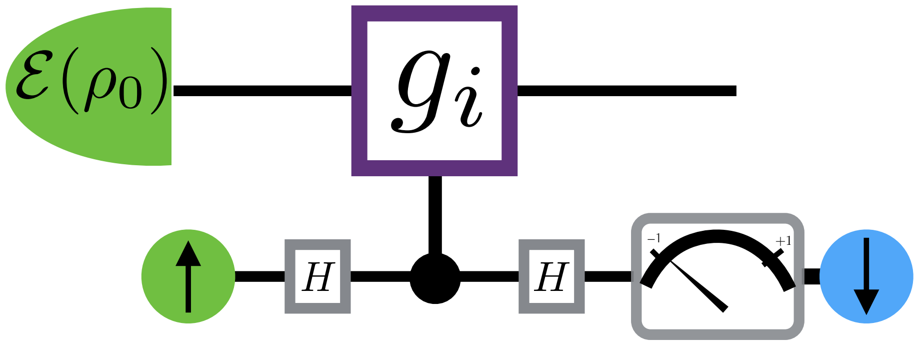
Perfect Entangler:

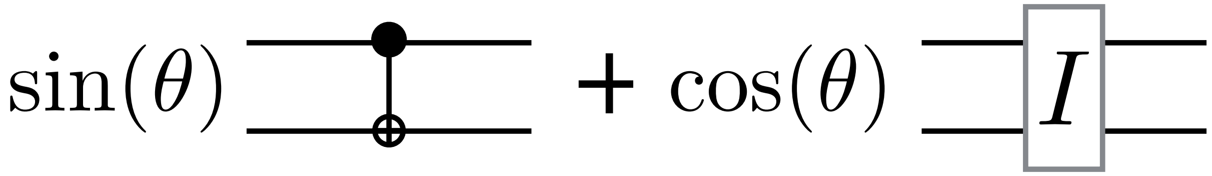


vs.

Partial Entangler:



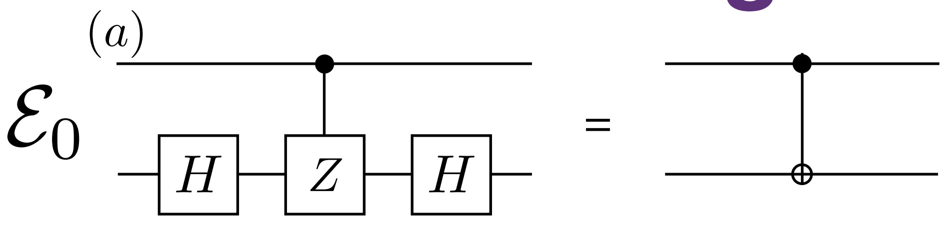




$$\chi^{(CX_{12}(\theta))} = \frac{1}{8}\begin{array}{cccc|c} \mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX & \\ 3\cos(\theta)+5 & -\cos(\theta)+2i\sin(\theta)+1 & -\cos(\theta)+2i\sin(\theta)+1 & \cos(\theta)-2i\sin(\theta)-1 & \mathbb{1}\mathbb{1} \\ -\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & Z\mathbb{1} \\ -\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & \mathbb{1}X \\ \cos(\theta)+2i\sin(\theta)-1 & \cos(\theta)-1 & \cos(\theta)-1 & 1-\cos(\theta) & ZX \end{array}$$

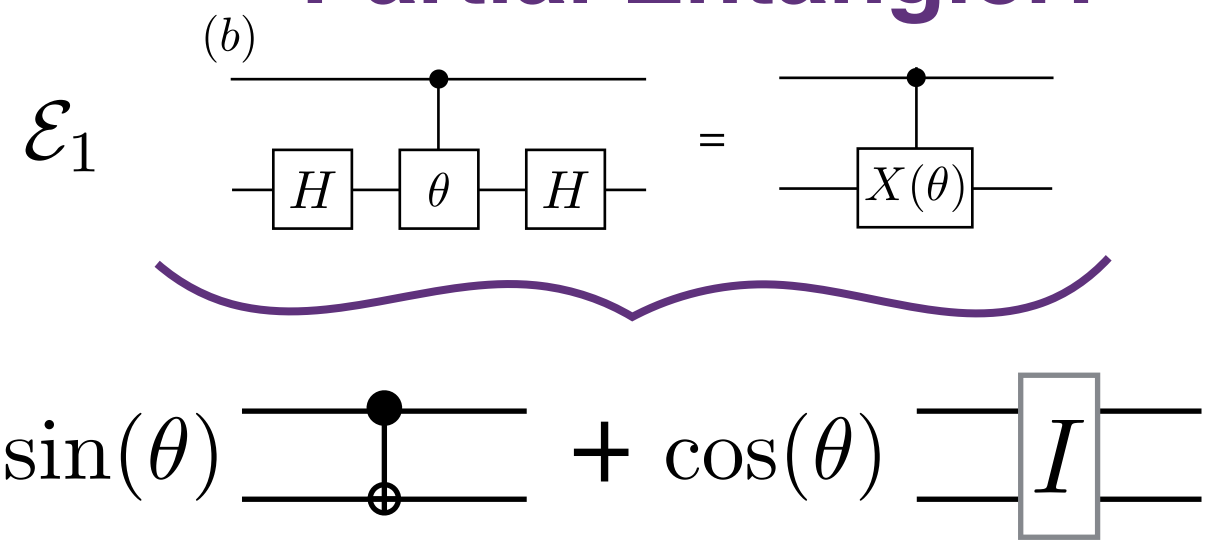
Quantum Channel Discrimination

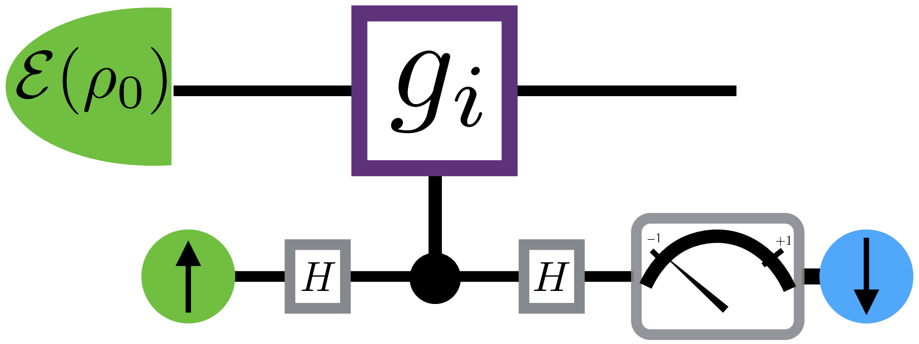
Perfect Entangler:



vs.

Partial Entangler:





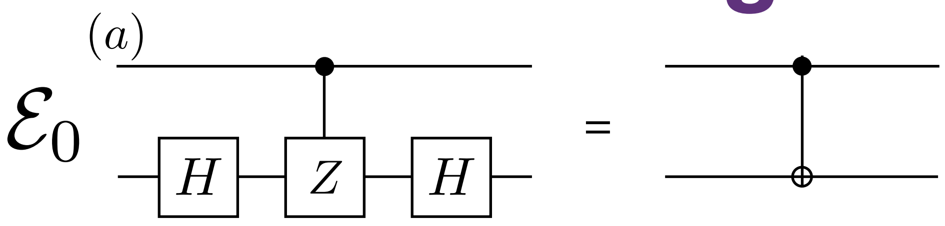
$$\chi^{(CX_{12}(\theta))} = \frac{1}{8}\begin{array}{c|c|c|c|c}
\mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX & \\
3\cos(\theta)+5 & -\cos(\theta)+2i\sin(\theta)+1 & -\cos(\theta)+2i\sin(\theta)+1 & \cos(\theta)-2i\sin(\theta)-1 & \mathbb{1}\mathbb{1} \\
-\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & Z\mathbb{1} \\
-\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & \mathbb{1}X \\
\cos(\theta)+2i\sin(\theta)-1 & \cos(\theta)-1 & \cos(\theta)-1 & 1-\cos(\theta) & ZX
\end{array}$$

Procedure:

1.Perform judicious stabilizer measurements $\hat{\chi}_{ij}$

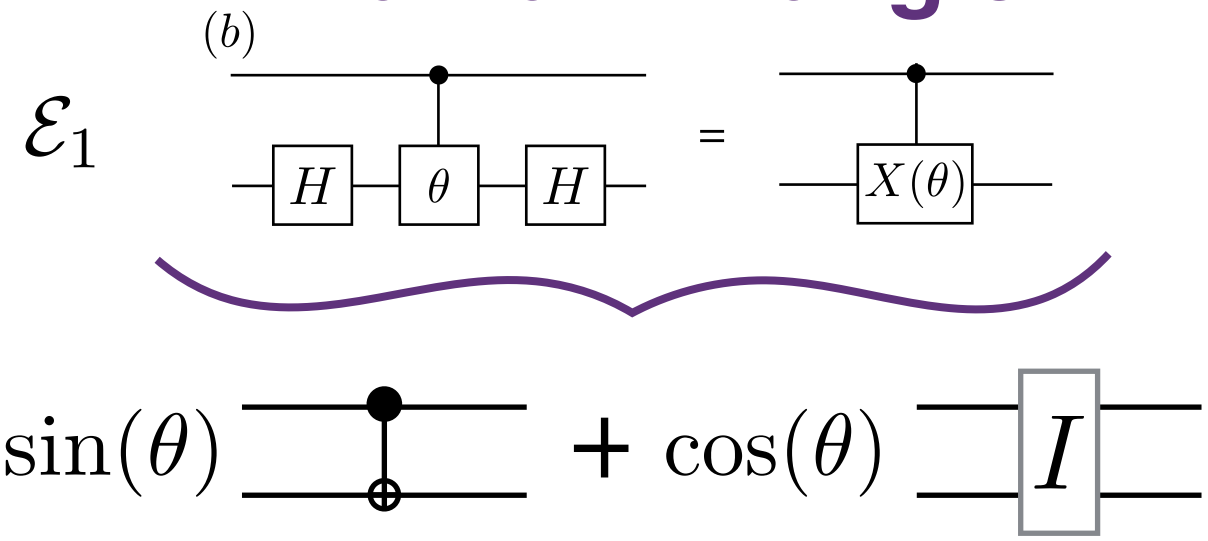
Quantum Channel Discrimination

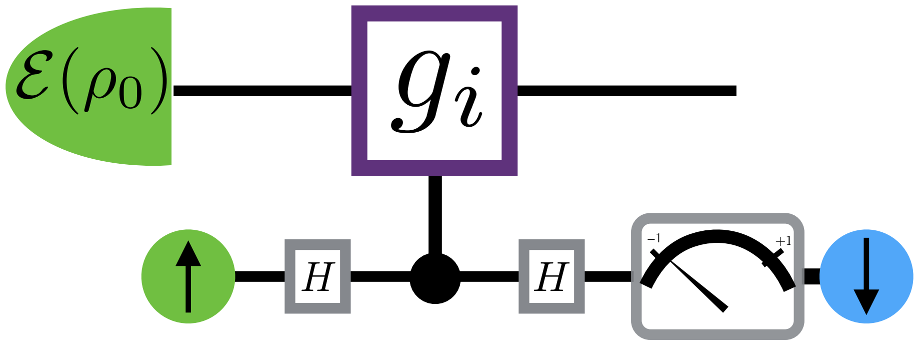
Perfect Entangler:



vs.

Partial Entangler:





$$\chi^{(CX_{12}(\theta))} = \frac{1}{8}\left(\begin{array}{cccc|c} \mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX & \\ 3\cos(\theta)+5 & -\cos(\theta)+2i\sin(\theta)+1 & -\cos(\theta)+2i\sin(\theta)+1 & \cos(\theta)-2i\sin(\theta)-1 & \mathbb{1}\mathbb{1} \\ -\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & Z\mathbb{1} \\ -\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & \mathbb{1}X \\ \cos(\theta)+2i\sin(\theta)-1 & \cos(\theta)-1 & \cos(\theta)-1 & 1-\cos(\theta) & ZX \end{array}\right)$$

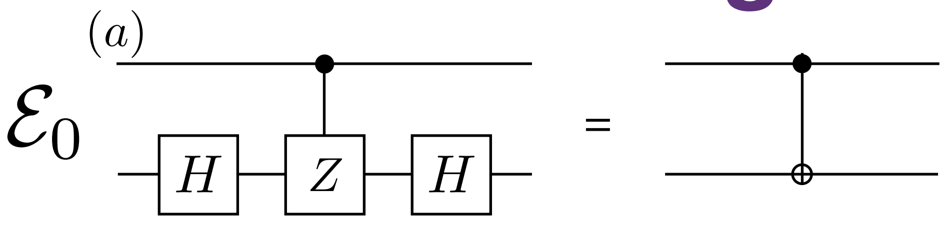
Procedure:

1. Perform judicious stabilizer measurements $\hat{\chi}_{ij}$
2. Estimate channel parameter $\hat{\chi}(\theta) \rightarrow \hat{\theta}$

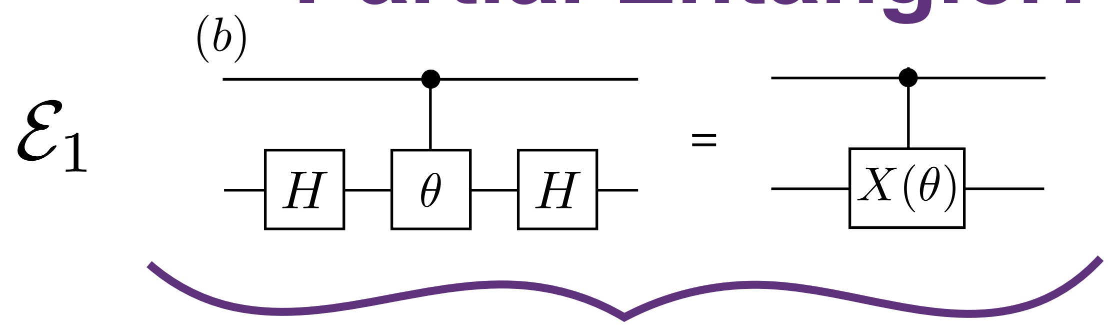
Quantum Channel Discrimination

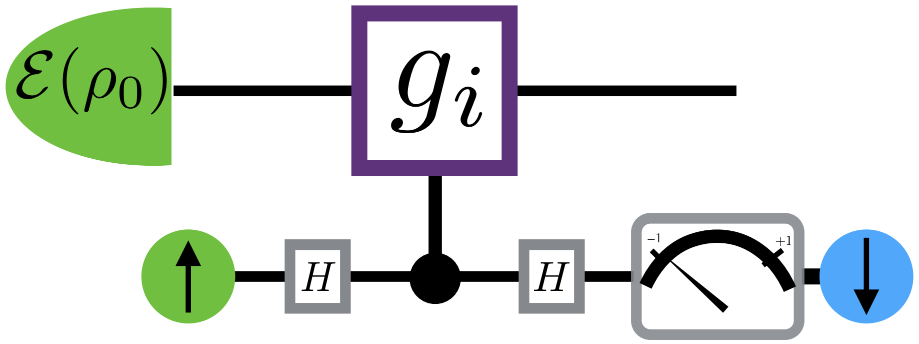
Perfect Entangler:

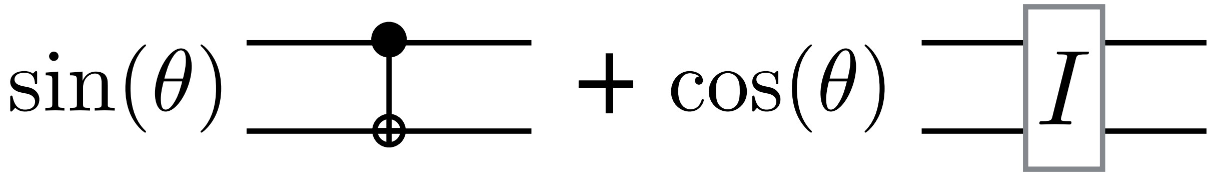
Partial Entangler:



vs.







$$\chi^{(CX_{12}(\theta))} = \frac{1}{8}\begin{array}{cccccc} & \mathbb{1}\mathbb{1} & Z\mathbb{1} & \mathbb{1}X & ZX & \\ \left(\vphantom{X}\right. & 3\cos(\theta)+5 & -\cos(\theta)+2i\sin(\theta)+1 & -\cos(\theta)+2i\sin(\theta)+1 & \cos(\theta)-2i\sin(\theta)-1 & \left.\vphantom{X}\right) \mathbb{1}\mathbb{1} \\ & -\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & Z\mathbb{1} \\ & -\cos(\theta)-2i\sin(\theta)+1 & 1-\cos(\theta) & 1-\cos(\theta) & \cos(\theta)-1 & \mathbb{1}X \\ & \cos(\theta)+2i\sin(\theta)-1 & \cos(\theta)-1 & \cos(\theta)-1 & 1-\cos(\theta) & ZX \end{array}$$

Procedure:

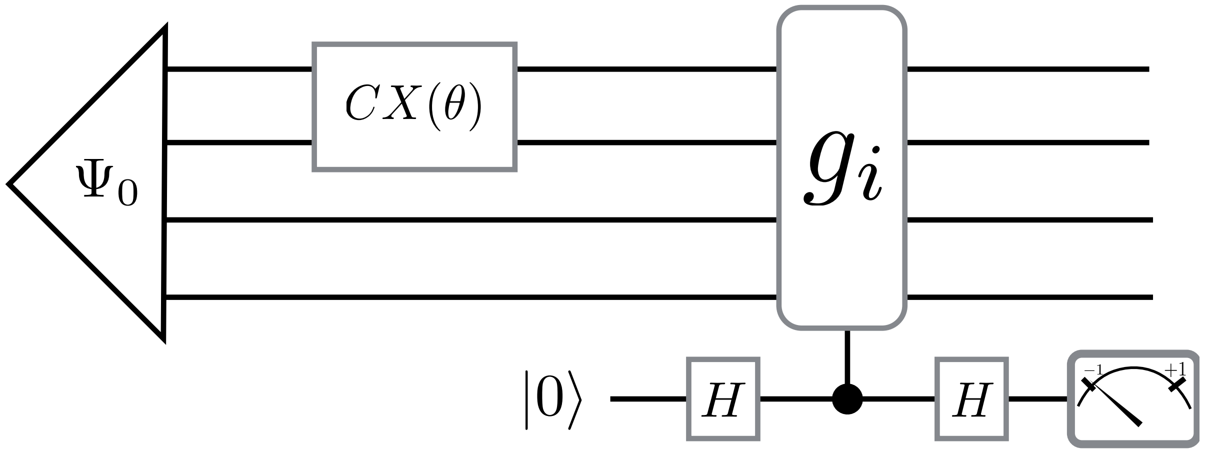
1.Perform judicious stabilizer measurements $\hat{\chi}_{ij}$

2.Estimate channel parameter $\hat{\chi}(\theta) \rightarrow \hat{\theta}$

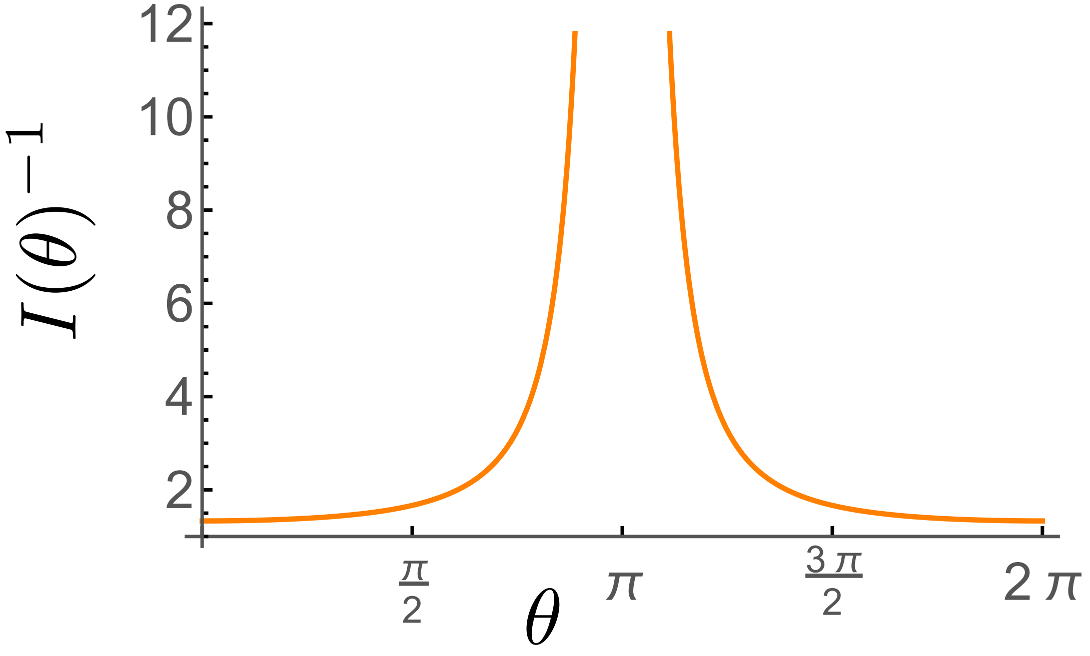
3.Statistical infer channel character $W(\hat{\theta}) \gtrless \lambda^*$

Statistical Inference

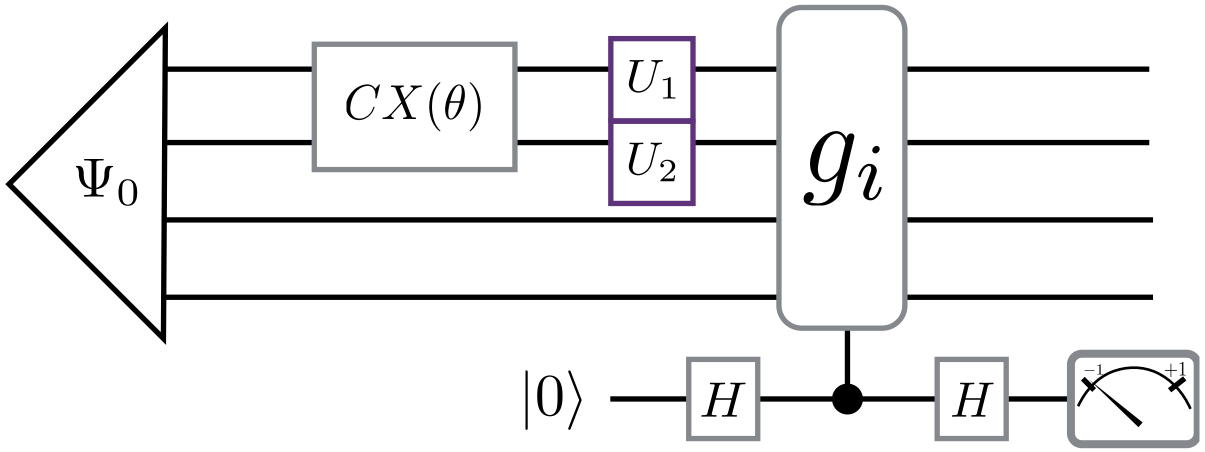
Statistical Inference



$$I(\theta) = -E[\frac{d}{d^2\theta}\log(\Pr(X|\theta))]$$



Statistical Inference

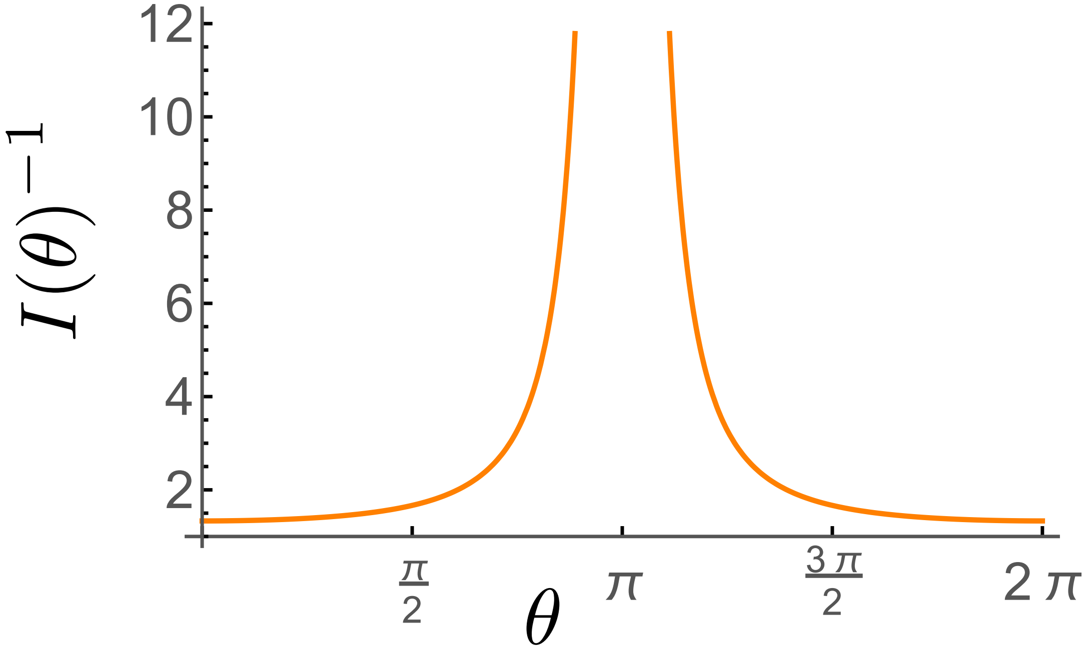


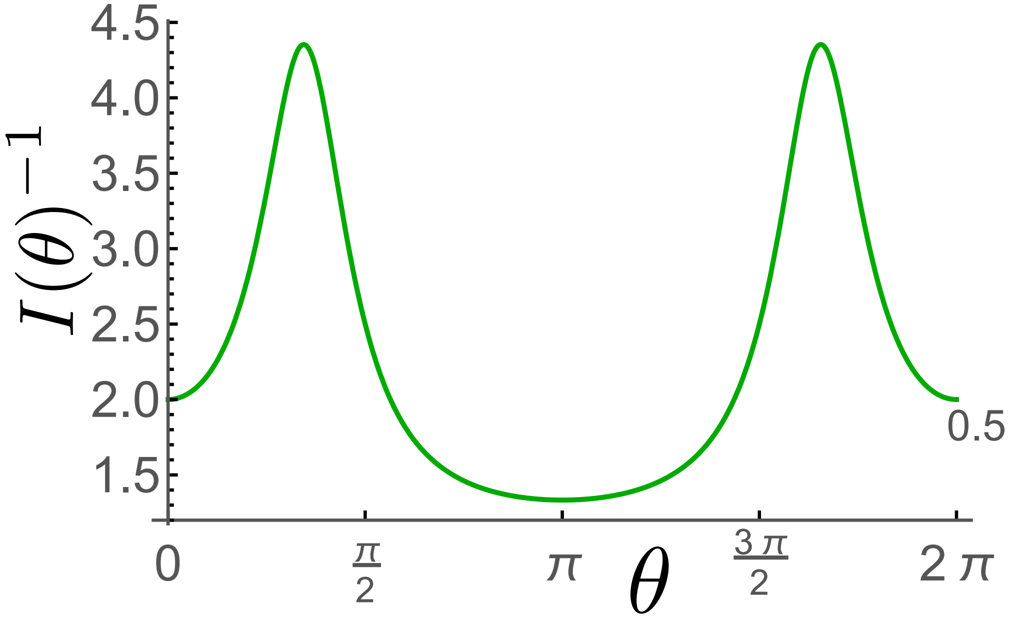
$$U_1 = (\mathbb{1} + iZ_1)/\sqrt{2}$$

$$U_2 = (\mathbb{1} + iX_2)/\sqrt{2}$$

Switch to new basis with better CRLB

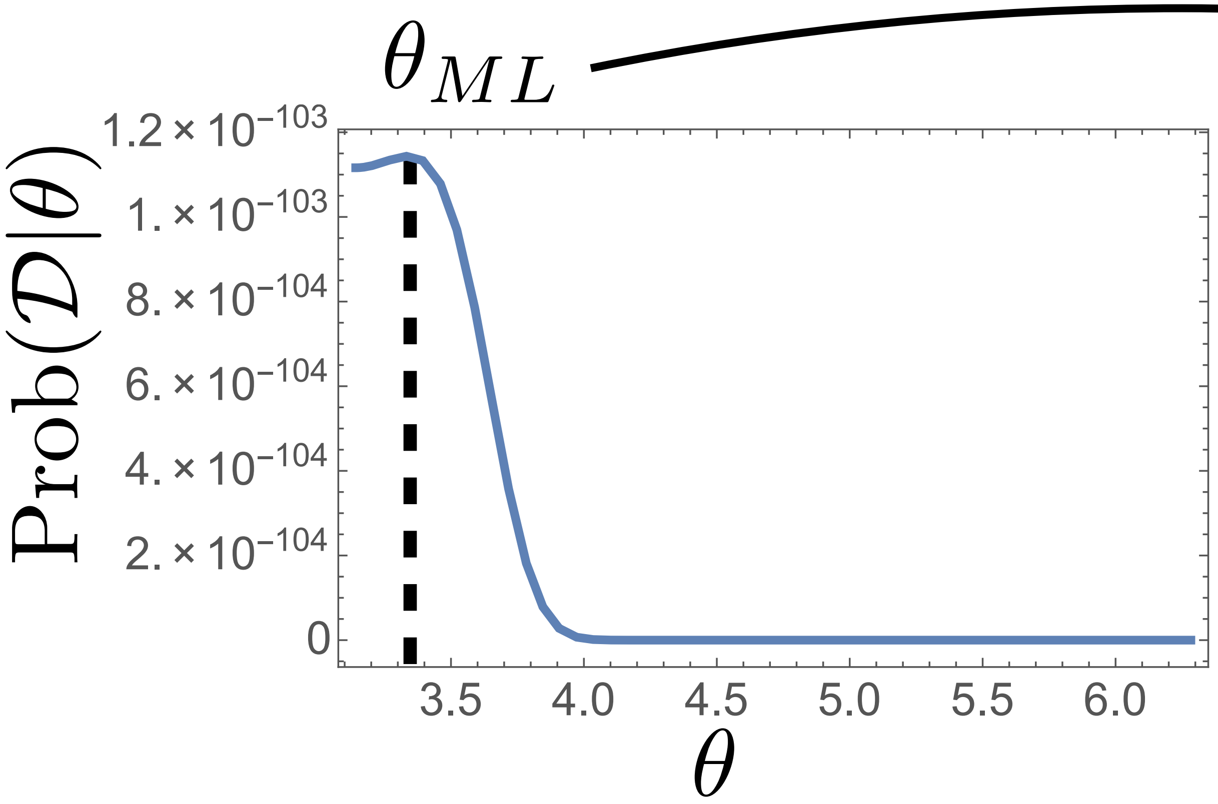
$$I(\theta) = -E[\frac{d}{d^2\theta}\log(\Pr(X|\theta))]$$





Statistical Inference

Statistical Inference

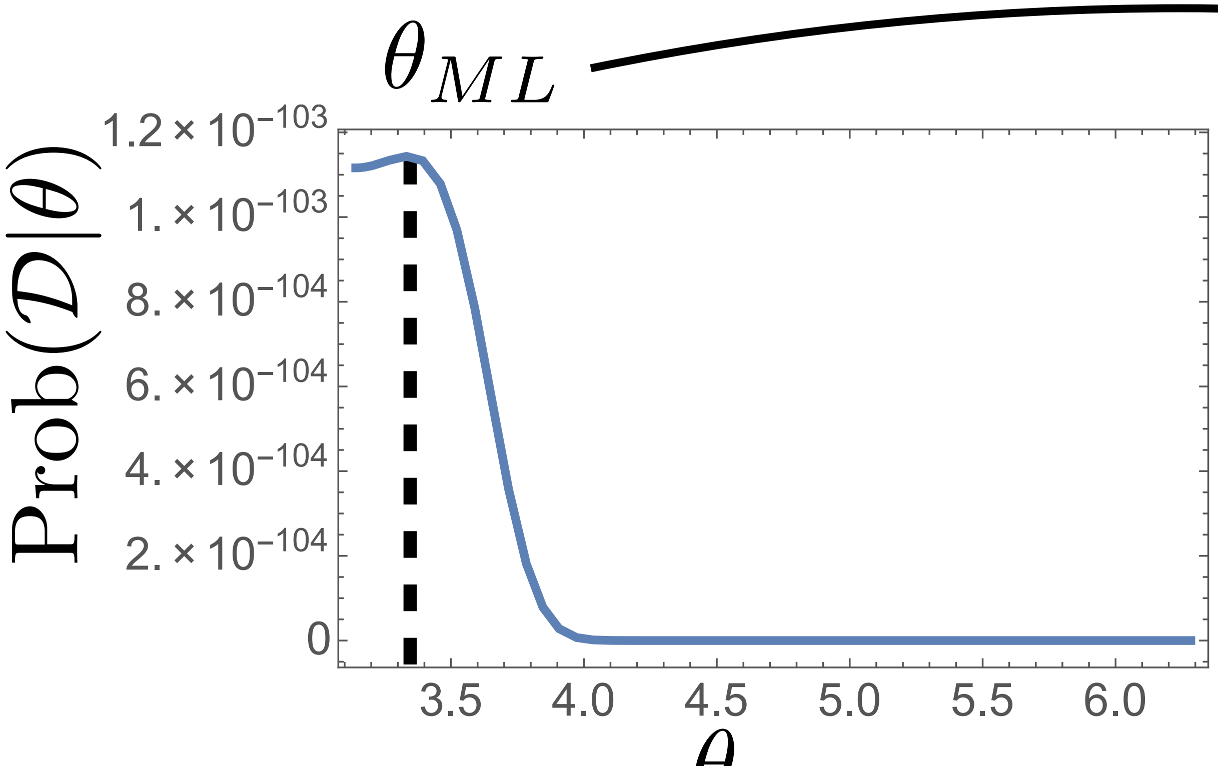


$$W = \frac{(\hat{\theta} - \theta_0)^2}{\mathrm{Var}(\hat{\theta})}$$

Wald Statistic

for channel discrimination

Statistical Inference

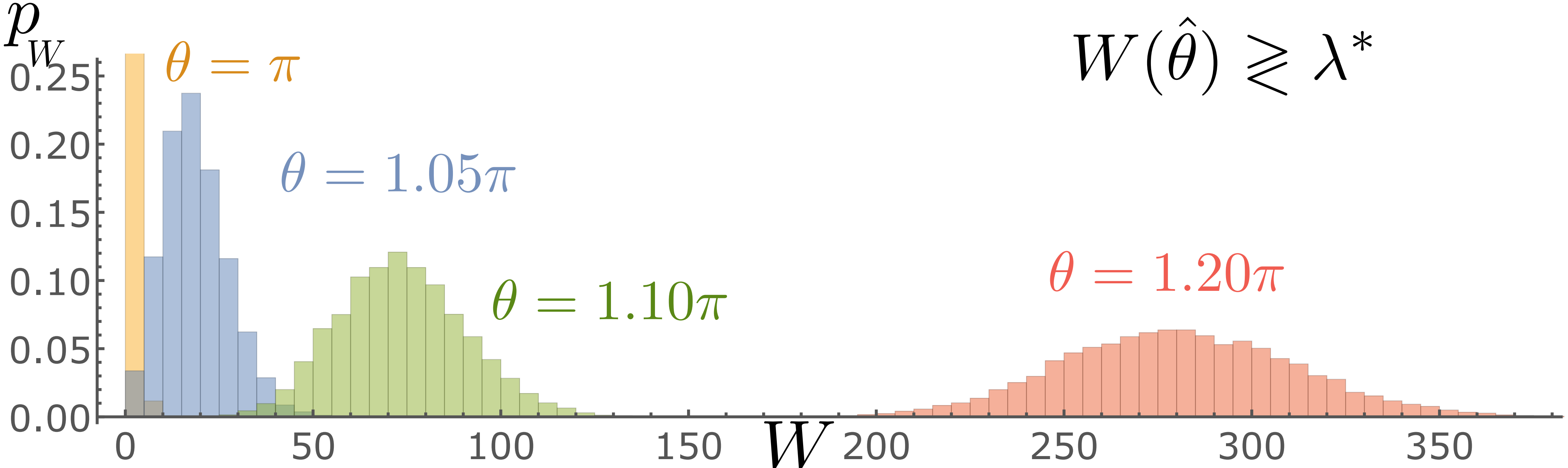


$$W = \frac{(\hat{\theta} - \theta_0)^2}{\mathrm{Var}(\hat{\theta})}$$

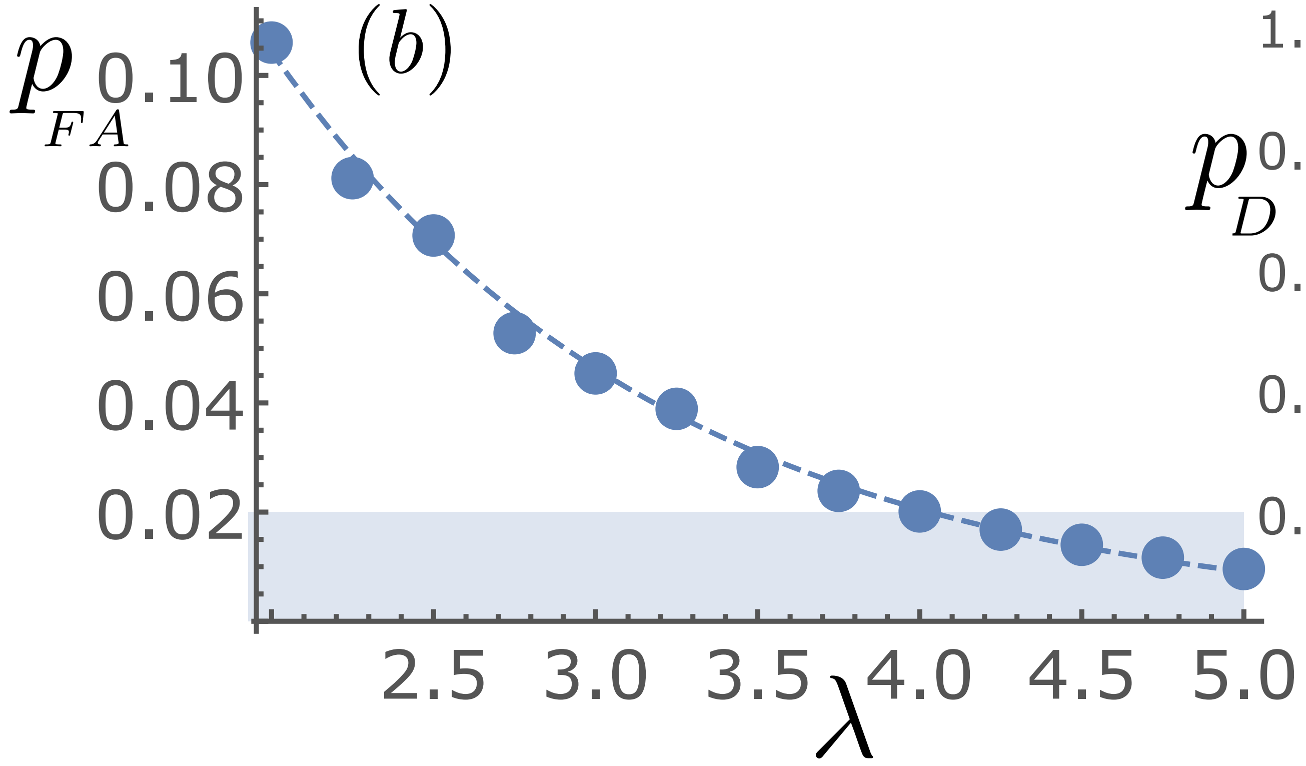
Wald Statistic

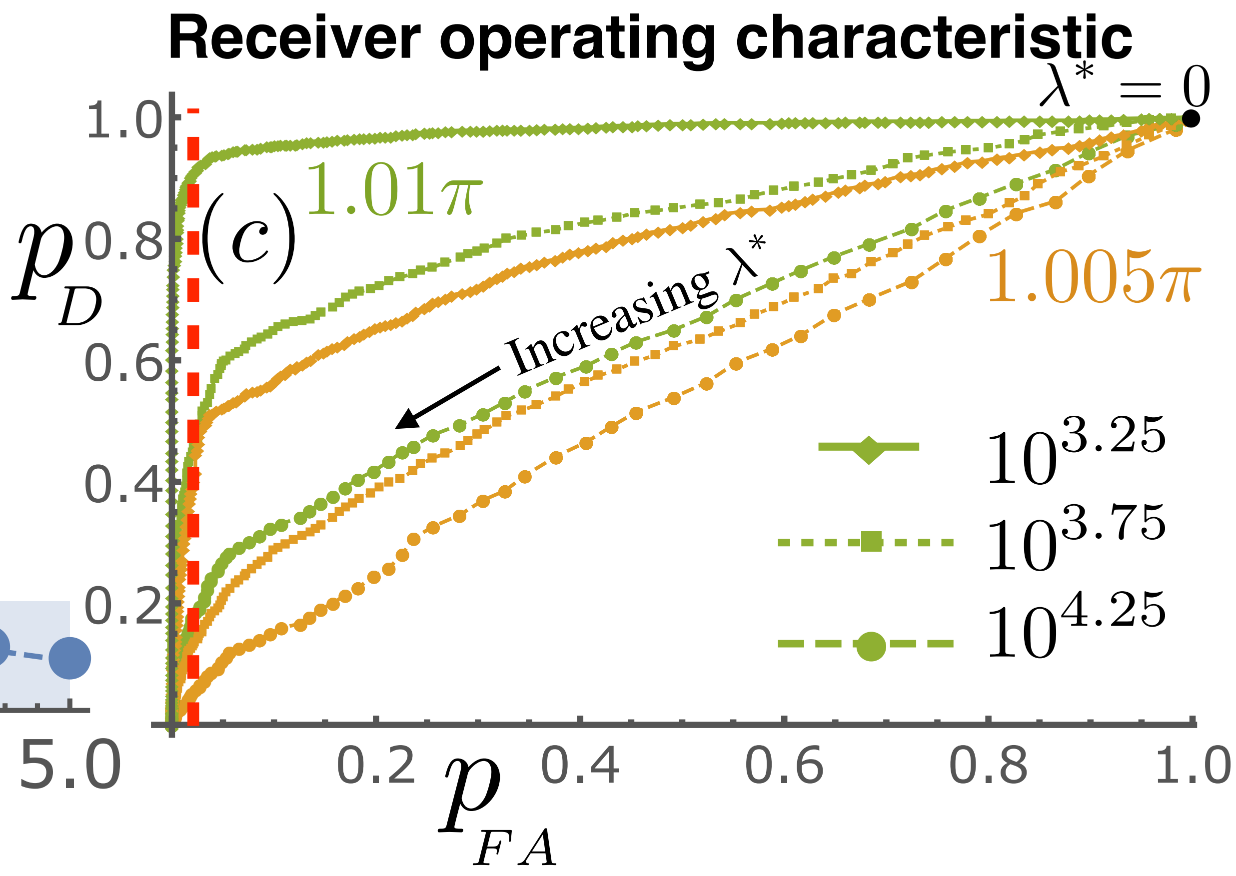
for channel discrimination

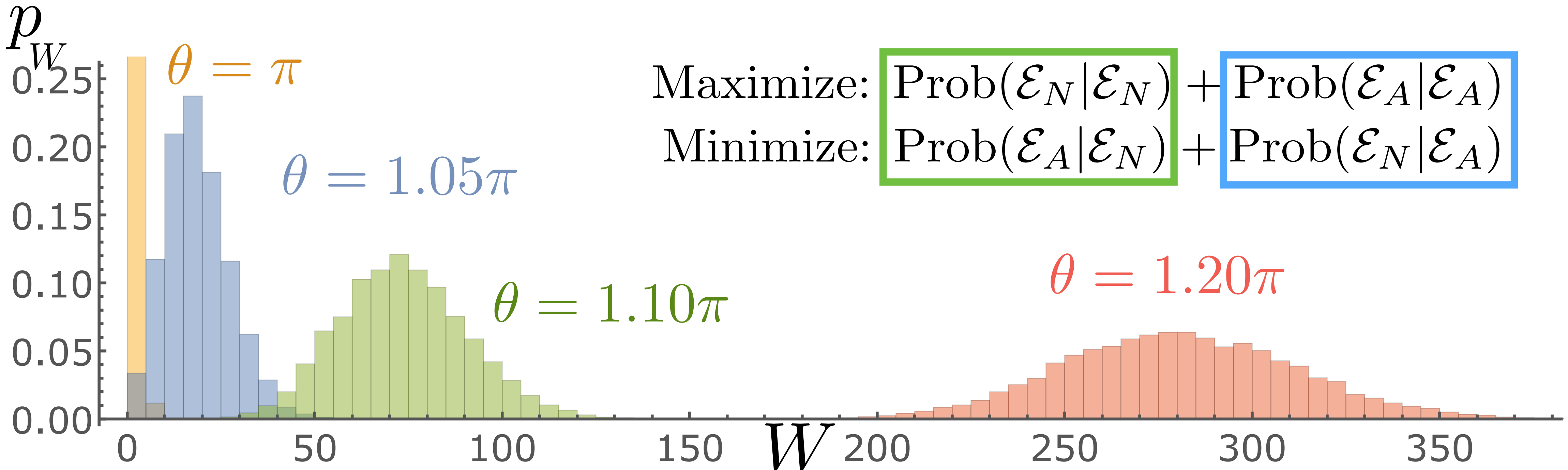
$$W(\hat{\theta}) \gtrless \lambda^*$$



Statistical Inference





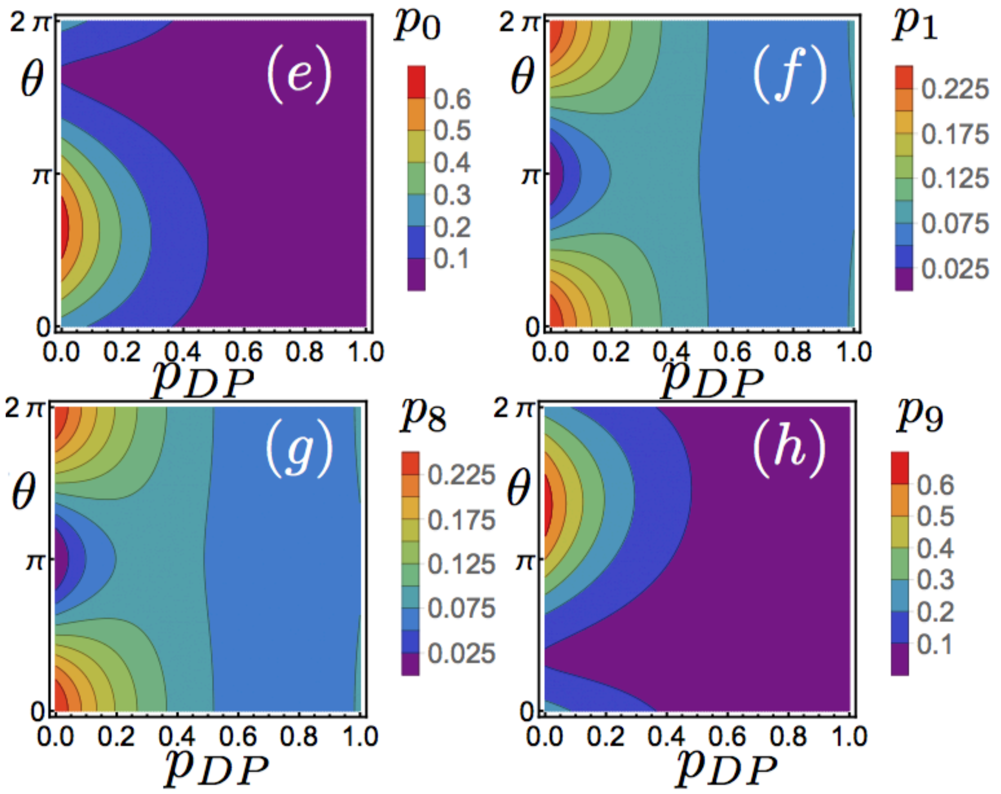


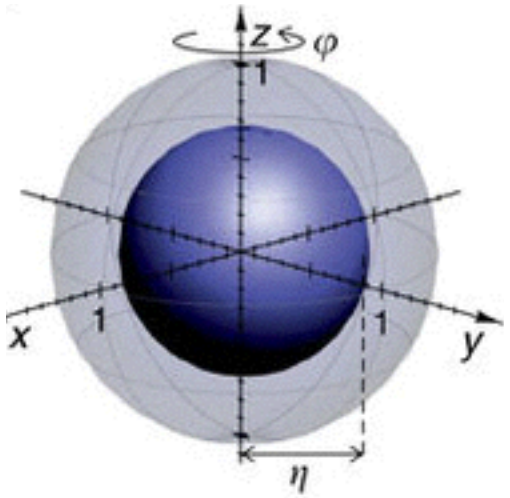
Maximize: $\text{Prob}(\mathcal{E}_N|\mathcal{E}_N) + \text{Prob}(\mathcal{E}_A|\mathcal{E}_A)$

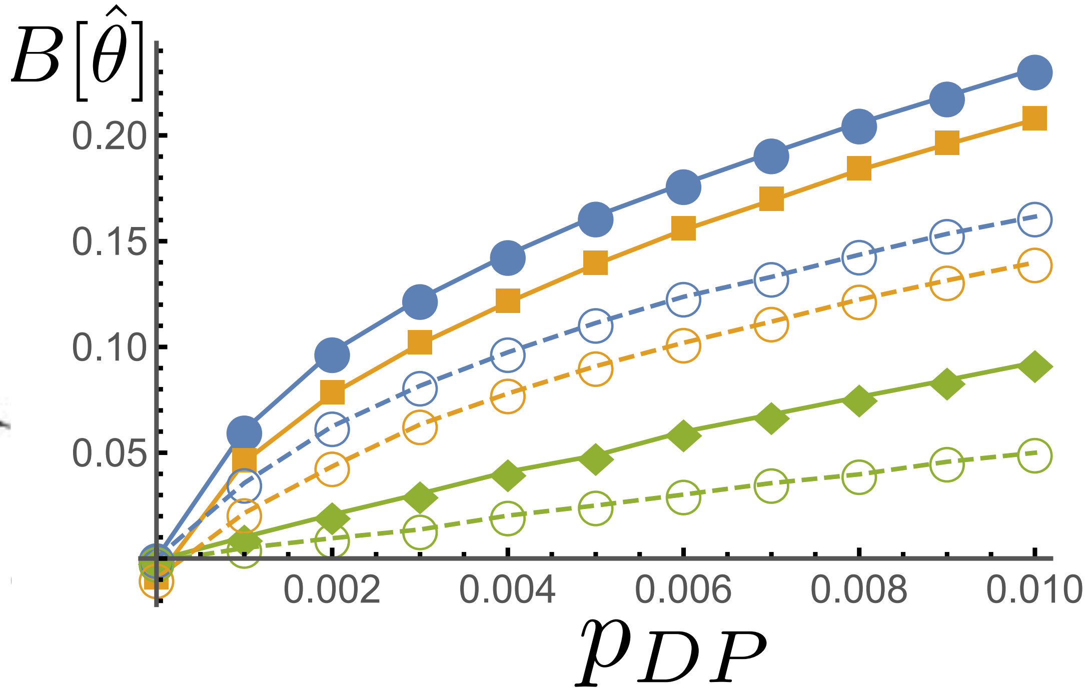
Minimize: $\text{Prob}(\mathcal{E}_A|\mathcal{E}_N) + \text{Prob}(\mathcal{E}_N|\mathcal{E}_A)$

Noisy parity checks

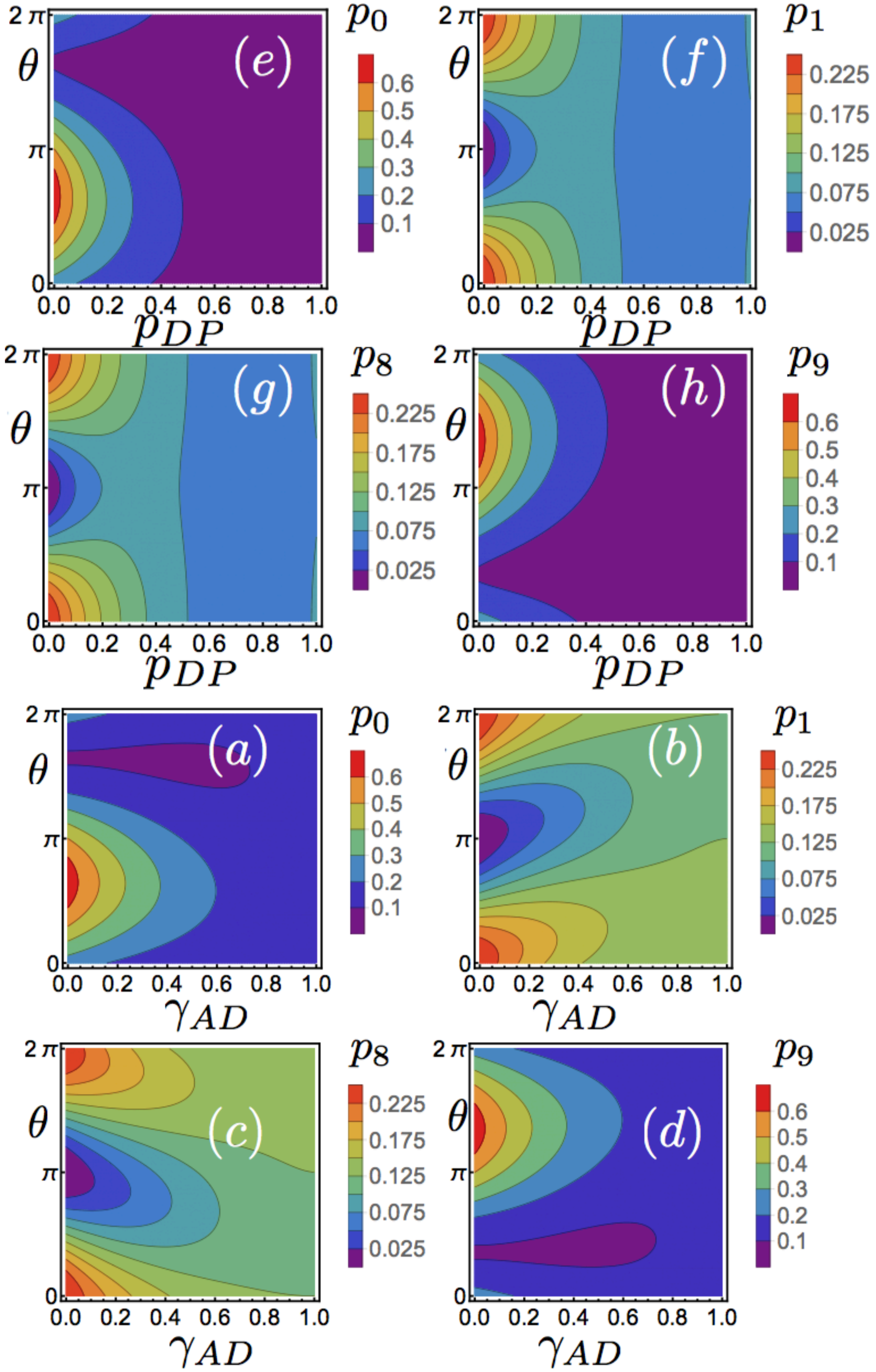
Noisy parity checks

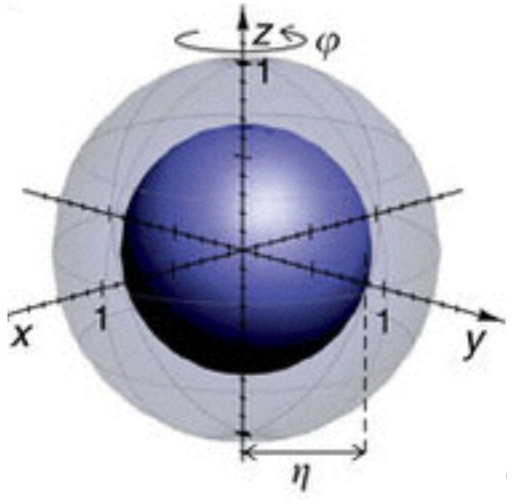


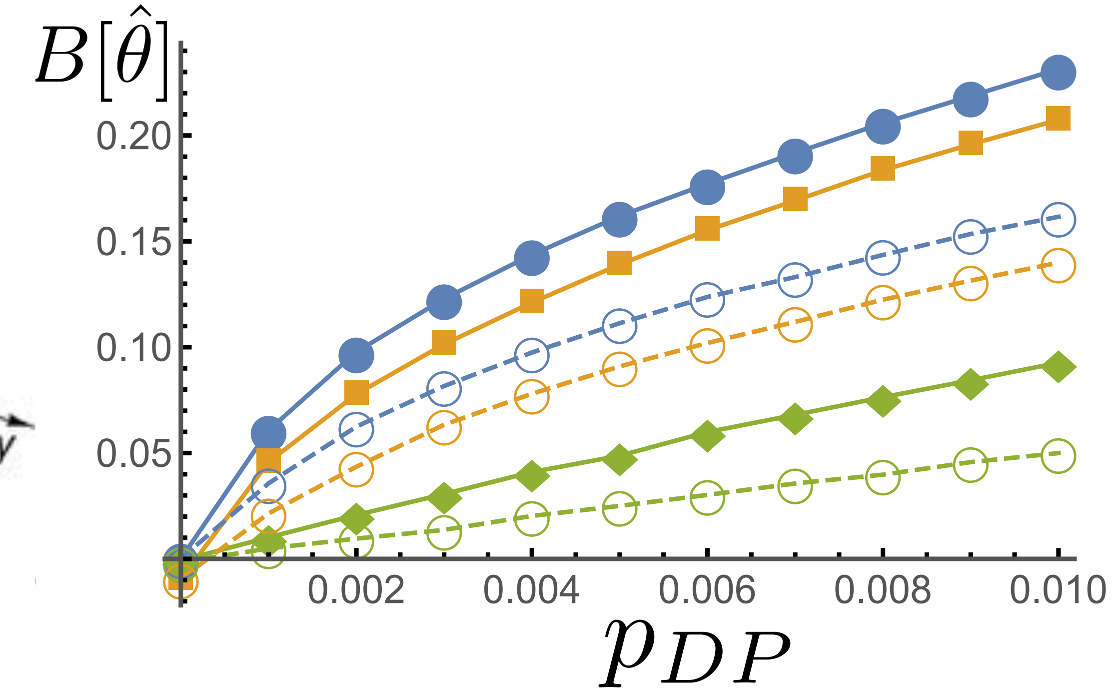


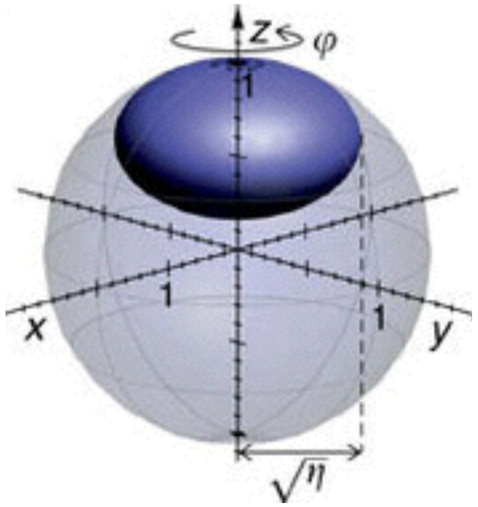


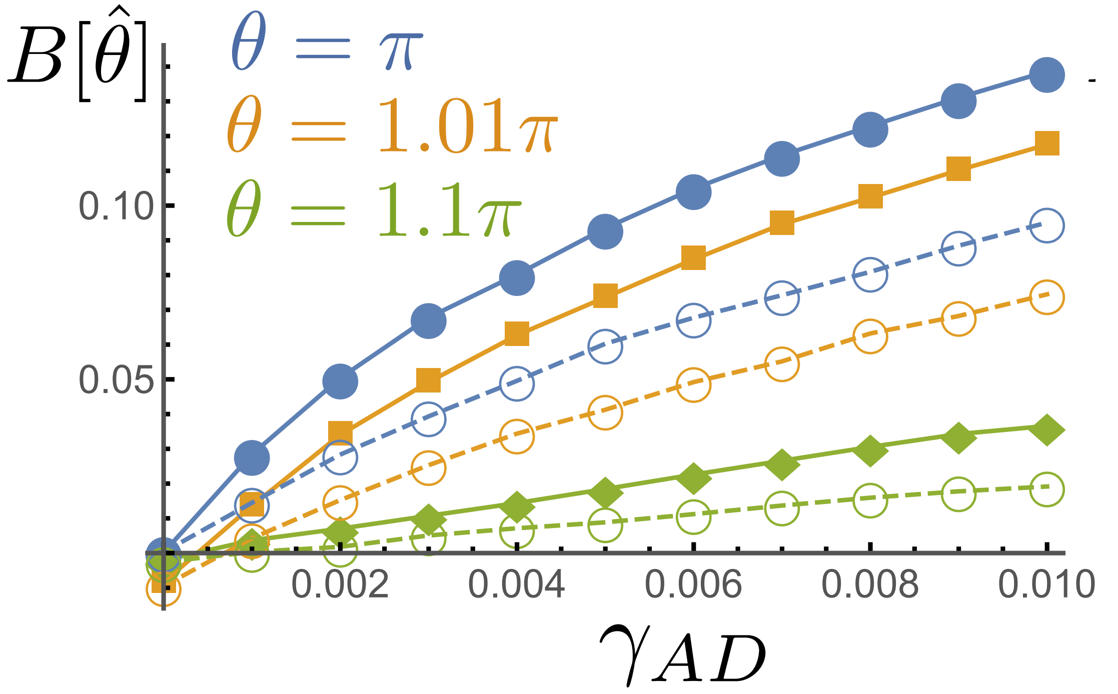
Noisy parity checks



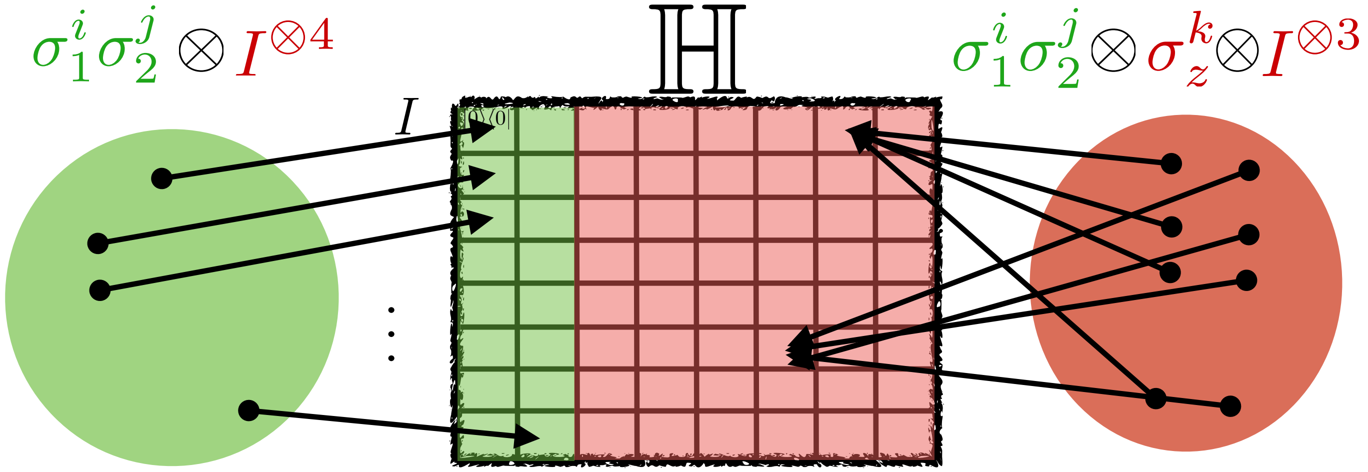


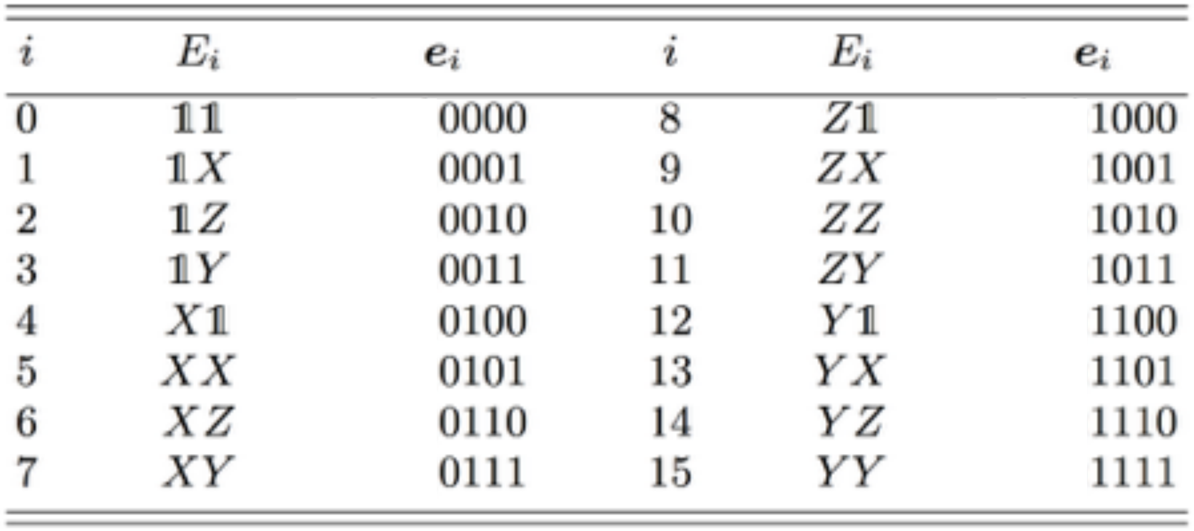






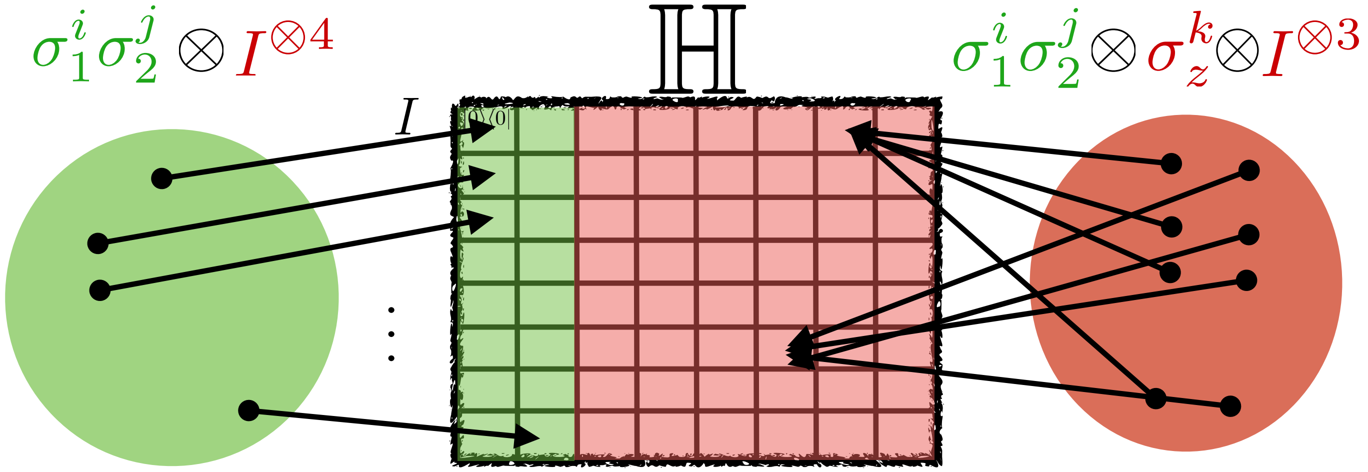
Selectivity Reduced Noise

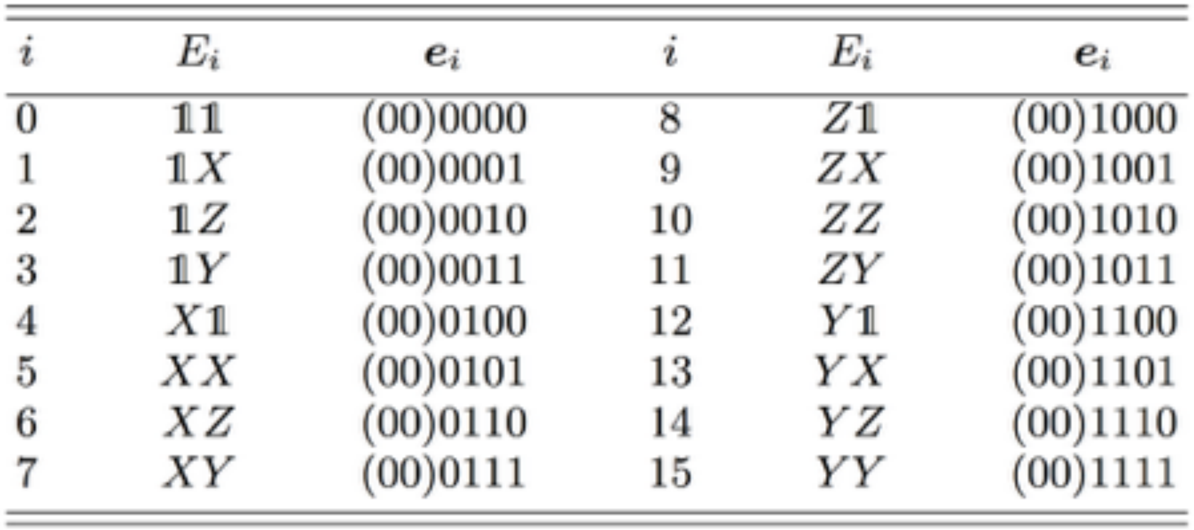




i	E_i	$\boldsymbol{e}_i$	i	E_i	$\boldsymbol{e}_i$
0	$\mathbb{1}\mathbb{1}$	0000	8	$Z\mathbb{1}$	1000
1	$\mathbb{1}X$	0001	9	ZX	1001
2	$\mathbb{1}Z$	0010	10	ZZ	1010
3	$\mathbb{1}Y$	0011	11	ZY	1011
4	$X\mathbb{1}$	0100	12	$Y\mathbb{1}$	1100
5	XX	0101	13	YX	1101
6	XZ	0110	14	YZ	1110
7	XY	0111	15	YY	1111

Selectivity Reduced Noise





i	E_i	$\mathbf{e}_i$	i	E_i	$\mathbf{e}_i$
0	$\mathbb{1}\mathbb{1}$	(00)0000	8	$Z\mathbb{1}$	(00)1000
1	$\mathbb{1}X$	(00)0001	9	ZX	(00)1001
2	$\mathbb{1}Z$	(00)0010	10	ZZ	(00)1010
3	$\mathbb{1}Y$	(00)0011	11	ZY	(00)1011
4	$X\mathbb{1}$	(00)0100	12	$Y\mathbb{1}$	(00)1100
5	XX	(00)0101	13	YX	(00)1101
6	XZ	(00)0110	14	YZ	(00)1110
7	XY	(00)0111	15	YY	(00)1111

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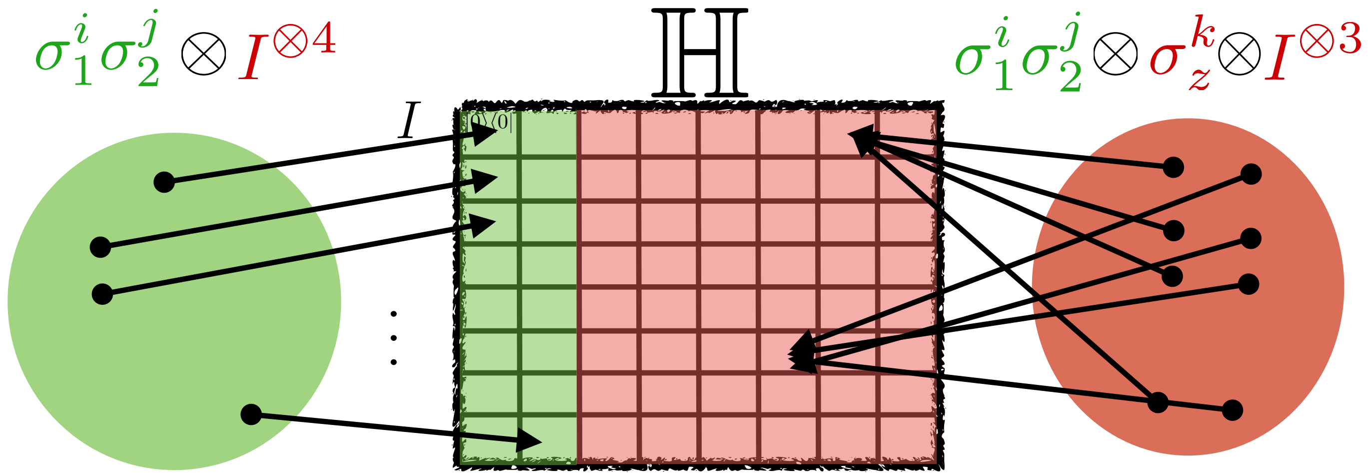

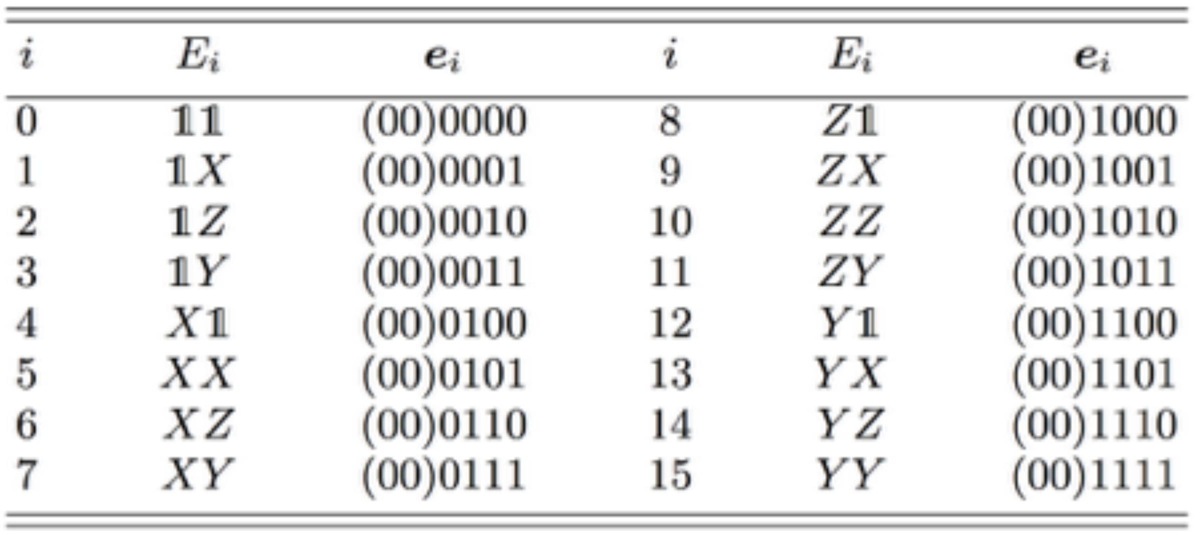

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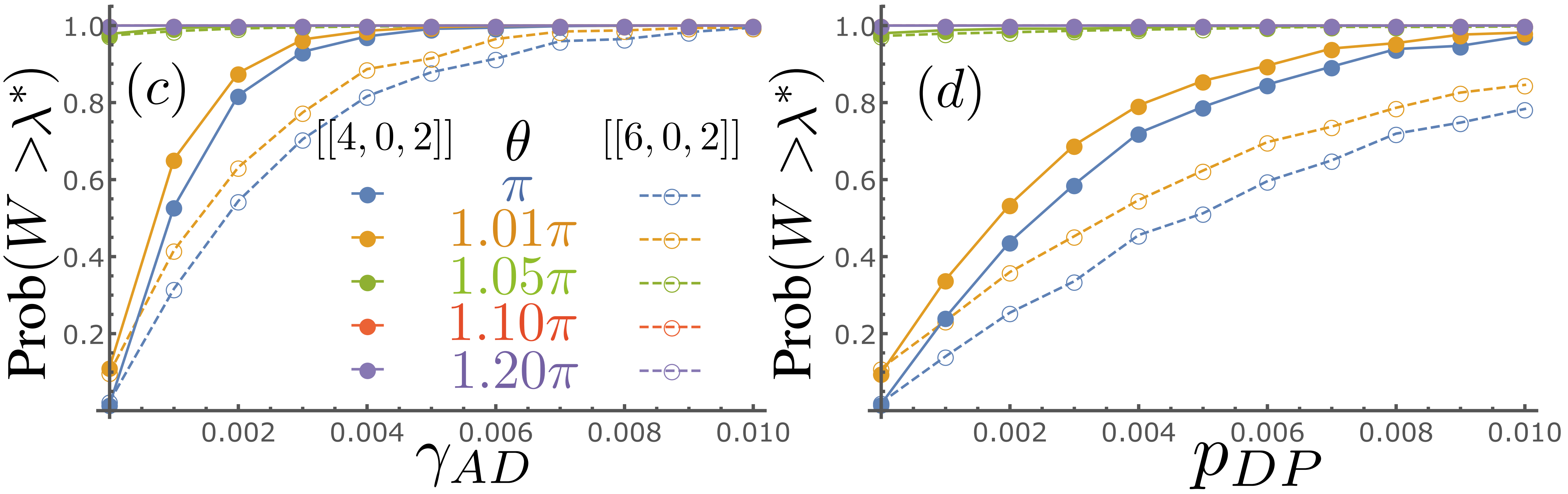

Model #2: Bit-flip Crosstalk

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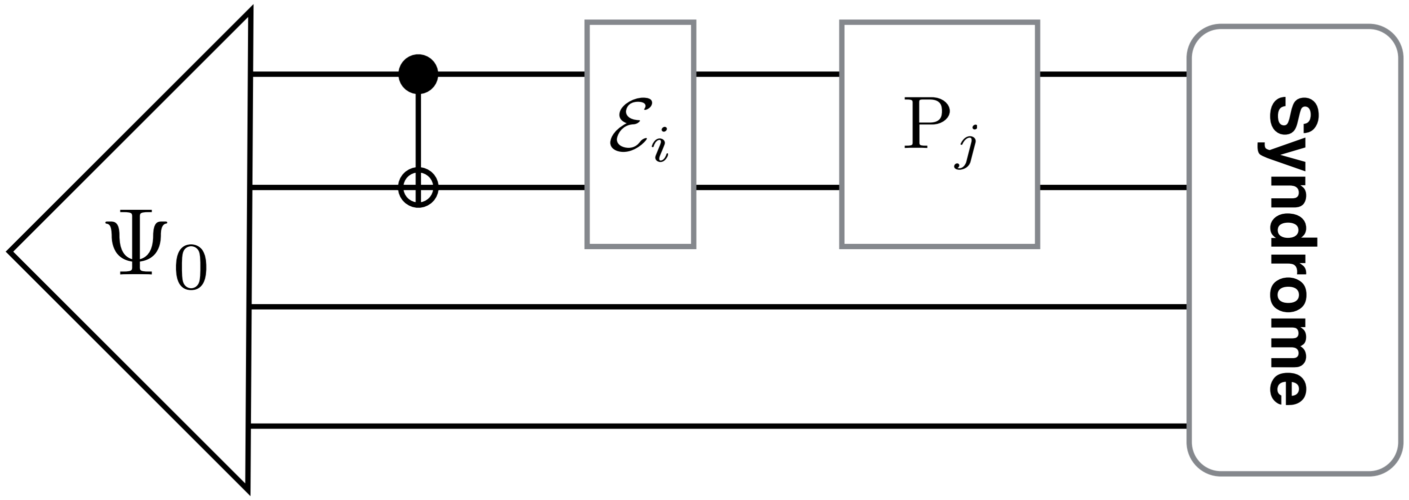

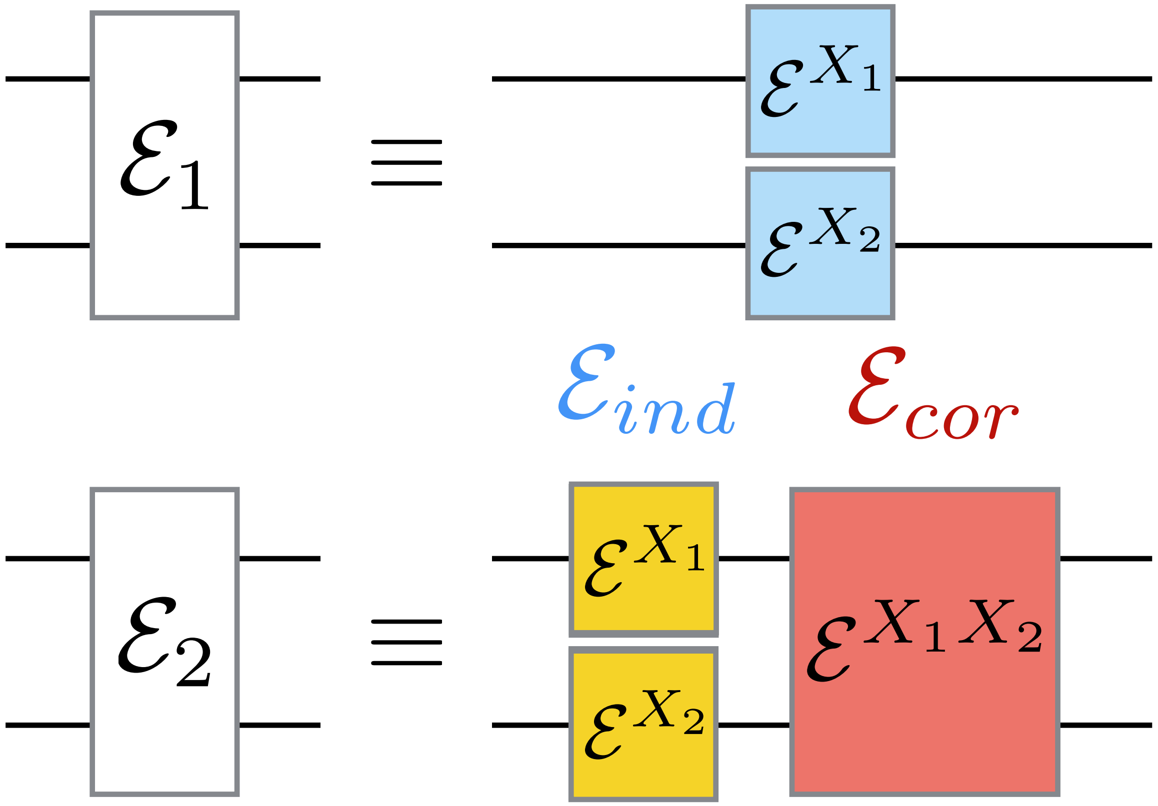

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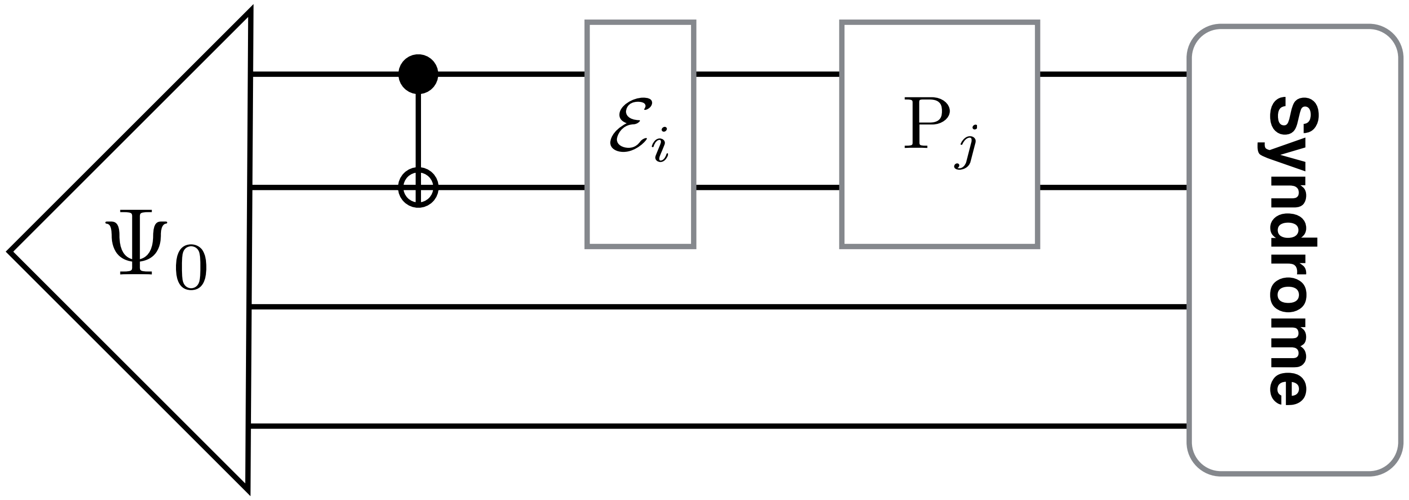
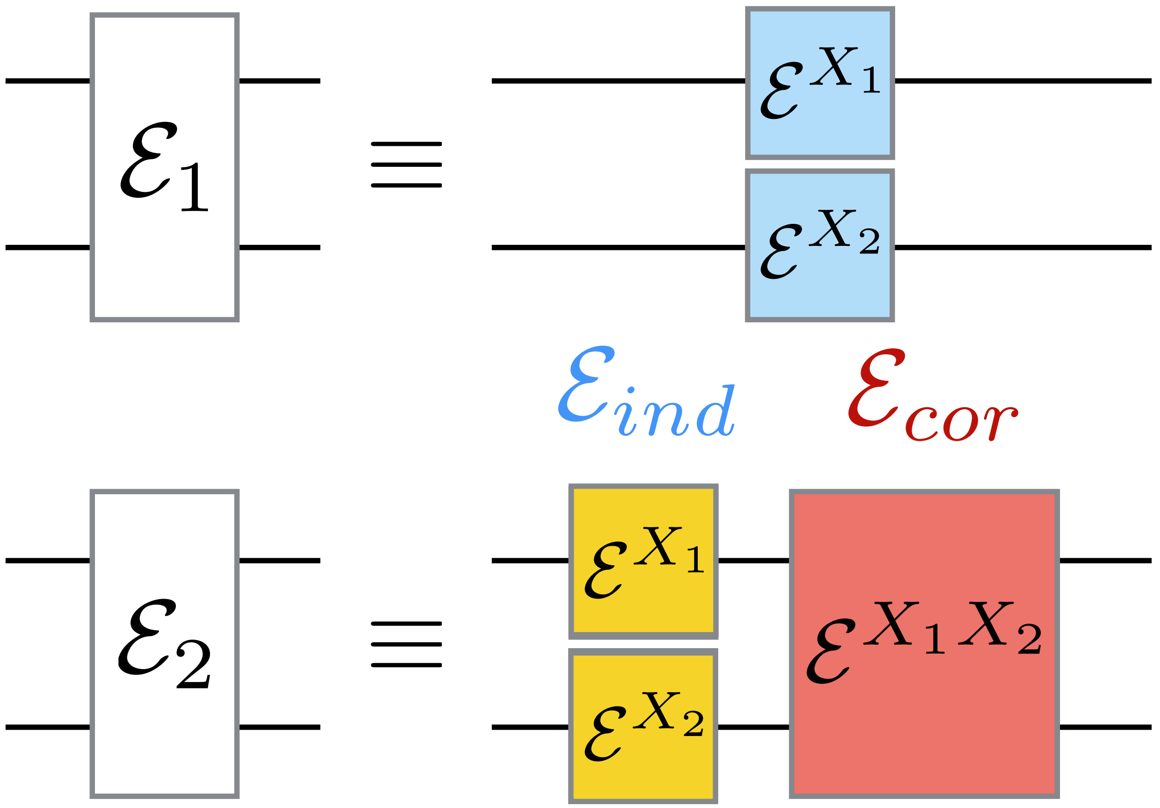


$$\mathcal{E}(\rho) = \sum_{m,n} \chi^{\uparrow}_{mn} F^{\uparrow}_{m} \rho F^{\uparrow\dagger}_{n} + \sum_{m',n'} \chi^{\downarrow}_{m'n'} F^{\downarrow}_{m'} \rho F^{\downarrow\dagger}_{n'}$$

$$\vec{F}^{\uparrow} = (II, ZI, IX, ZX)$$

$$\vec{F}^{\downarrow} = (XI, YI, XX, YX)$$



$$\chi^{\uparrow} = \frac{1}{4}\begin{pmatrix} \alpha+\beta & \beta-\alpha & \alpha+\beta & \alpha-\beta \\ \beta-\alpha & \alpha+\beta & \beta-\alpha & -\alpha-\beta \\ \alpha+\beta & \beta-\alpha & \alpha+\beta & \alpha-\beta \\ \alpha-\beta & -\alpha-\beta & \alpha-\beta & \alpha+\beta \end{pmatrix}$$

$$\chi^{\downarrow} = \frac{1}{4}\begin{pmatrix} \gamma+\delta & \gamma+\delta & \gamma-\delta & \delta-\gamma \\ \gamma+\delta & \gamma+\delta & \gamma-\delta & \delta-\gamma \\ \gamma-\delta & \gamma-\delta & \gamma+\delta & -(\gamma+\delta) \\ \delta-\gamma & \delta-\gamma & -(\gamma+\delta) & \gamma+\delta \end{pmatrix}$$

$$\alpha = p_1^X p_2^X p_{12}^X + (1-p_1^X)(1-p_2^X)(1-p_{12}^X)$$
$$\beta = \left(1-p_1^X\right)\left(1-p_{12}^X\right) p_2^X + p_1^X \left(1-p_2^X\right) p_{12}^X$$
$$\gamma = \left(1-p_2^X\right)\left(1-p_{12}^X\right) p_1^X + \left(1-p_1^X\right) p_2^X p_{12}^X$$
$$\delta = p_1^X \left(1-p_{12}^X\right) p_2^X + \left(1-p_1^X\right)\left(1-p_2^X\right) p_{12}^X$$

Bit Flip Inference

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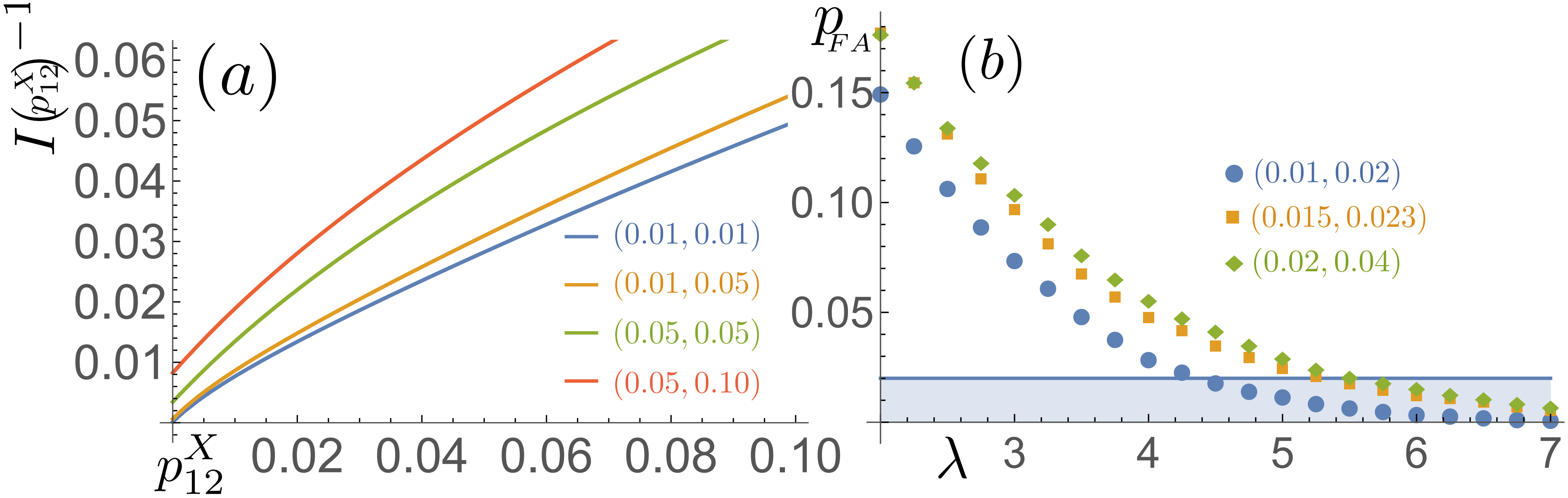
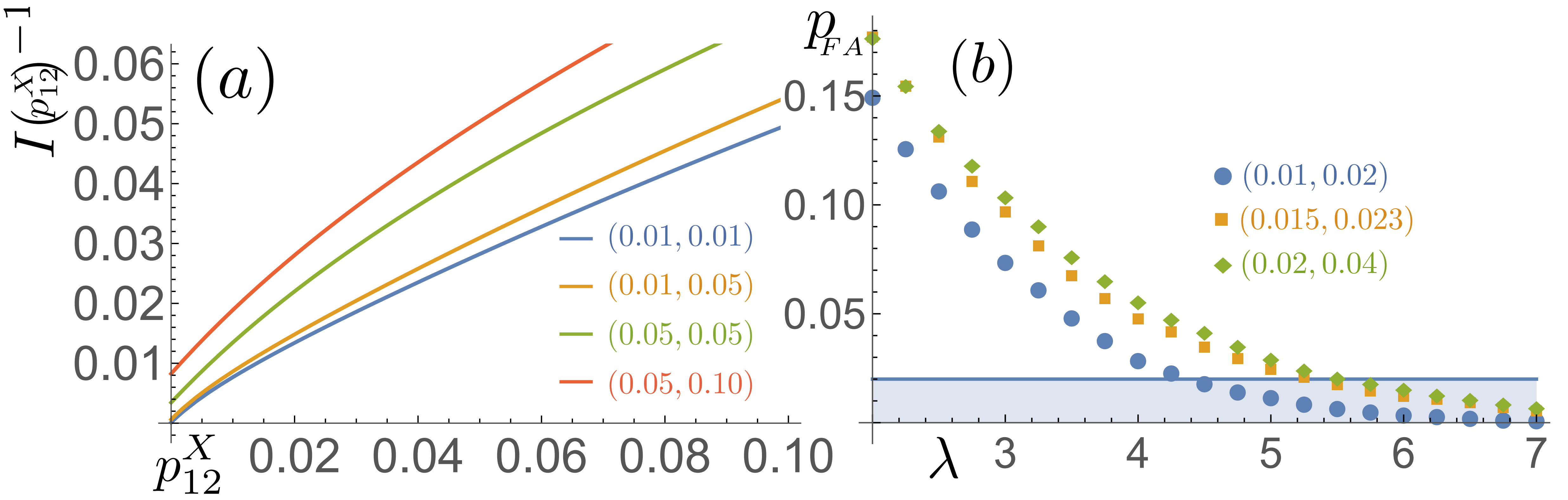
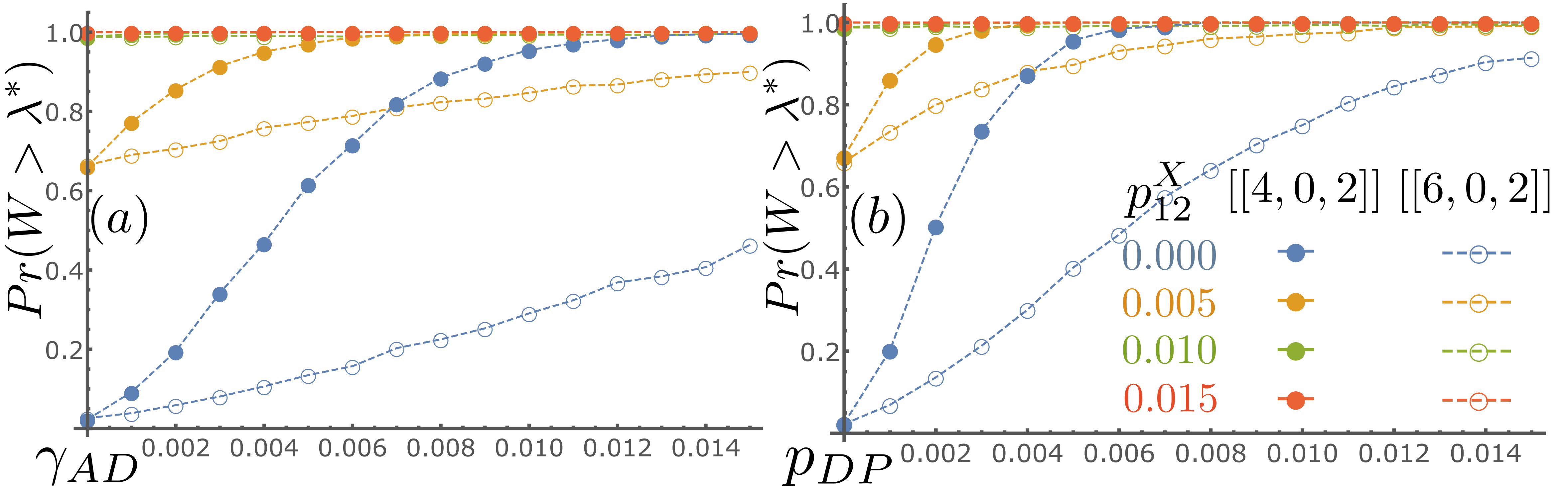


Bit Flip Inference





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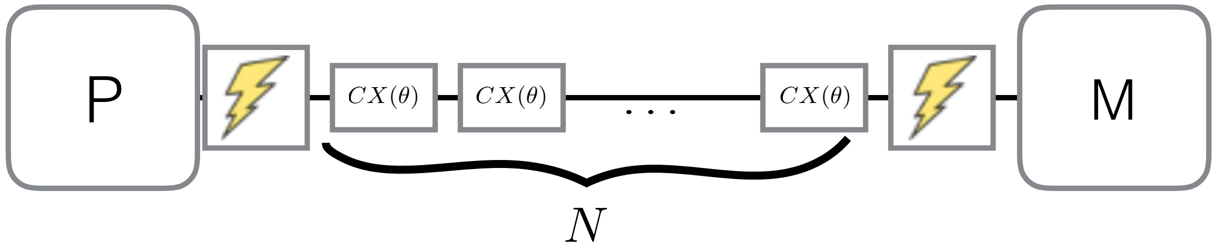
Conclusions

1. Developed direct process tomography quantum codes with error **detecting** codes with non-degenerate/degenerate bi-partite structure
2. Implemented statistical channel discrimination protocols by **selectively** probing relevant process elements
3. We'd like to study if one can further develop sensing with quantum codes by also implementing adaptive (code) strategies? Relation to compressed sensing?
4. Develop SPAM error resistant protocol by studying discrimination characteristics when state subjected to a growing number of channel instances



Thank you!