Universality of transport coefficients in the Haldane-Hubbard model

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Joint work with V. Mastropietro, M. Porta and I. Jauslin

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Outline

1. Overview
2. Introduction
3. The model and the main results
4. Sketch of the proof

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- Model: Haldane-Hubbard, simplest interacting Chern insulator.
  Several approximate and numerical results available.
  Very few (if none) rigorous results.

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- Method: constructive Renormalization Group +
  + lattice symmetries + Ward Identities + Schwinger-Dyson

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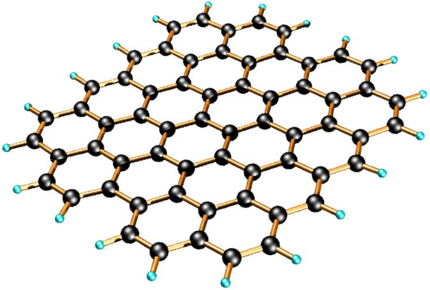
Graphene

Graphene is a 2D allotrope of carbon: single layer of graphite.



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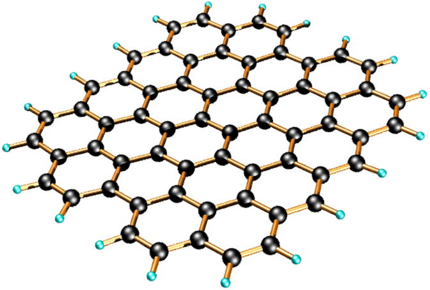

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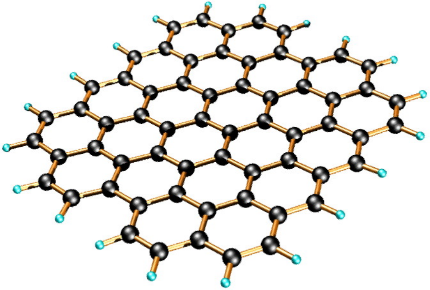

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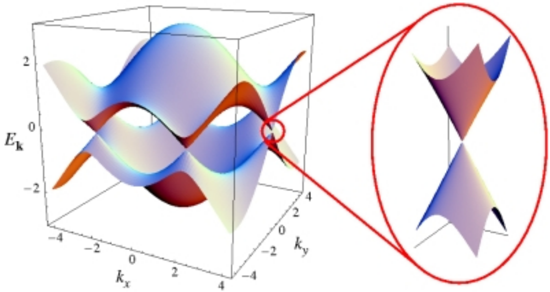
Graphene and graphene-like materials have unusual, and remarkable, mechanical and electronic transport properties.

Here we shall focus on its transport properties.

Graphene

Peculiar transport properties due to its unusual band structure:

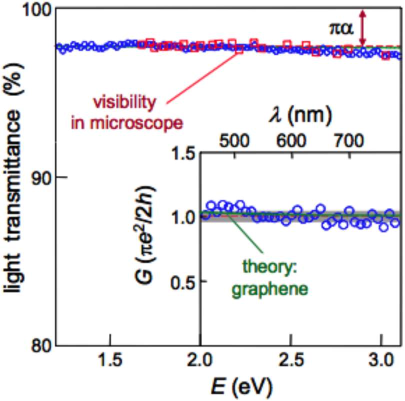
- at half-filling the Fermi surface degenerates into two Fermi points
- Low energy excitations: 2D massless Dirac fermions ($v \simeq c/300$) $\Rightarrow$ 'semi-metallic' QED-like behavior at non-relativistic energies



Minimal conductivity

Signatures of the relativistic nature of quasi-particles:

1. Minimal conductivity at zero charge carriers density.
   Measurable at $T = 20^o$ C from $t(\omega) = \frac{1}{(1+2\pi\sigma(\omega)/c)^2}$

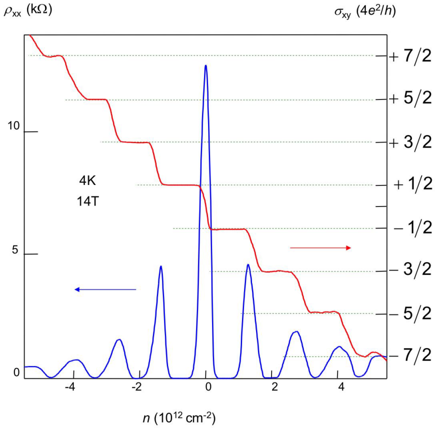


For clean samples and
$k_B T \ll \hbar\omega \ll bandwidth$,

$$\sigma(\omega) = \sigma_0 = \frac{\pi}{2}\frac{e^2}{h}$$

Anomalous QHE

2. Constant transverse magnetic field: anomalous IQHE.
   Shifted plateaus: $\sigma_{12} = 4\frac{e^2}{h}(N + \frac{1}{2})$:



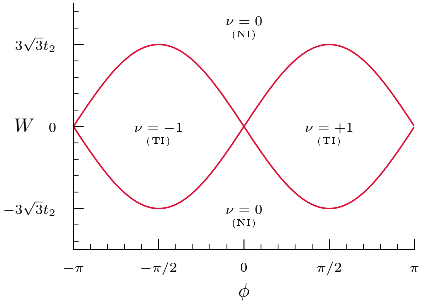
Observable at $T = 20^o$.

At low temperatures: plateaus measured at $\sim 5 \times 10^{-11}$ precision.

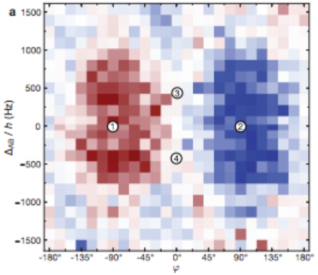
QHE without net magnetic flux

3. Another unusual setting for IQHE with zero net magnetic flux: proposal by Haldane in 1988 (Nobel prize 2016). Main ingredients:

- dipolar magnetic field ⇒ n-n-n hopping t_2 acquires complex phase
- staggered potential on the sites of the two sub-lattices



Phase diagram (predicted...)



(... and measured, Esslinger et al. '14)

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- QHE: let $P_\mu = \chi(H \le \mu) =$ Fermi proj. If $\mathbf{E}|P_\mu(x;y)| \le Ce^{-c|x-y|}$, i.e., $\mu \in$ spectral gap, or $\mu \in$ mobility gap:

$$\sigma_{12} = \frac{ie^2}{\hbar} \mathrm{Tr}\, P_\mu[[X_1, P_\mu], [X_2, P_\mu]] \in \frac{e^2}{h} \cdot \mathbb{Z}$$

(Thouless-Kohmoto-Nightingale-Den Nijs '82, Avron-Seiler-Simon '83, '94, Bellissard-van Elst-Schulz Baldes '94, Aizenman-Graf '98...)

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- Minimal conductivity: gapless, semi-metallic, ground state. Exact computation in a model of free Dirac fermions (Ludwig-Fisher-Shankar-Grinstein '94), or in tight binding model (Stauber-Peres-Geim '08).

Effects of interactions?

What are the effects of electron-electron interactions? In graphene, interaction strength is intermediate/large:

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- Minimal longitudinal conductivity: no geometrical interpretation. Cancellations due to Ward Identities? Big debate in the graphene community, still ongoing (Mishchenko, Herbut-Juričić-Vafek, Sheehy-Schmalian, Katsnelson et al., Rosenstein-Lewkowicz-Maniv ...)

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Today: Universality of σ_{12} (up to the critical line) and of σ_{11} (on the critical line) in the weakly interacting Haldane-Hubbard model.

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Note: our method: no topology/geometry, no assumption on gap: decay of interacting correlations + cancellations from WI and SD.

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The lattice and the Hamiltonian

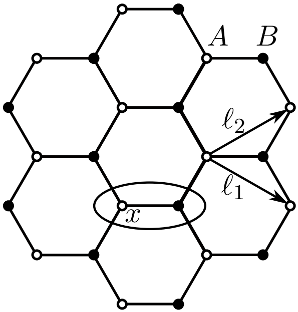

Figure: Dimer $\rightsquigarrow (a^{\pm}_{x,\sigma},\, b^{\pm}_{x,\sigma})$.

- Hamiltonian: $\mathcal{H} = \mathcal{H}_0 + U\mathcal{V}$, where

$$\mathcal{H}_0 = \text{n.n.} + \text{complex n.n.n. hopping} + \text{ staggered potential} - \mu\mathcal{N}$$

$$\mathcal{V} = \sum_x (n^A_{x,\uparrow} n^A_{x,\downarrow} + n^B_{x,\uparrow} n^B_{x,\downarrow})$$

Conductivity

- Finite temperature, finite volume Gibbs state (eventually, $\beta, L \to \infty$):

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- Kubo formula: linear response at $t = 0$, after having switched on adiabatically a weak external field $e^{\eta t} E$ at $t = -\infty$

The non-interacting Hamiltonian (Haldane model)

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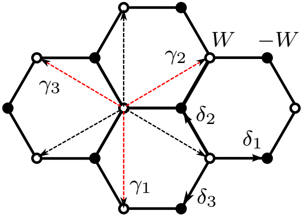

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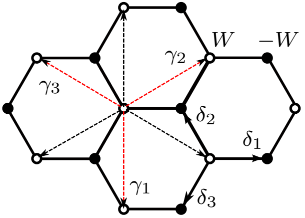

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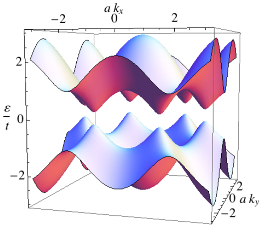
- Gapped system. Gaps:

$$\Delta_\pm = |m_\pm|\,,\qquad m_\pm = W\pm 3\sqrt{3}t_2\sin\phi.$$

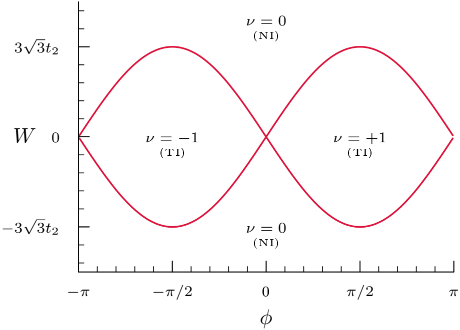
$=$ "mass" of Dirac fermions.



Non-interacting phase diagram

- If $U = 0$, μ is kept in between the two bands, and $m_\pm \neq 0$:

$$\sigma_{12} = \frac{2e^2}{h}\nu \,, \qquad \nu = \frac{1}{2}[\mathrm{sgn}(m_-) - \mathrm{sgn}(m_+)]$$



- Simplest model of topological insulator.
  Building brick for more complex systems (*e.g.* Kane-Mele model).

Phase transitions in the Haldane-Hubbard model

Theorem (Giuliani, Jauslin, Mastropietro, Porta 2016)

There exists $U_0 > 0$ and a function ("renormalized mass")

$$m_{R,\omega} = m_\omega + F_\omega(m_\pm; U) \quad \textit{where} \quad F_\omega = O(U), \quad \omega = \pm$$

such that, for $U \in (-U_0, U_0)$, choosing $\mu = \mu(m_\pm; U)$:

$$\sigma_{12} = \frac{e^2}{h}[\mathrm{sgn}(m_{R,-}) - \mathrm{sgn}(m_{R,+})], \quad \text{if} \quad m_{R,\pm} \neq 0,$$

$$\sigma_{ii}^{cr} := \sigma_{ii}\big|_{m_{R,\omega}=0} = \frac{e^2}{h}\frac{\pi}{4}, \quad \text{if} \quad m_{R,-\omega} \neq 0\,.$$

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Phase transitions in the Haldane-Hubbard model

Theorem (Giuliani, Jauslin, Mastropietro, Porta 2016)

There exists $U_0 > 0$ and a function ("renormalized mass")

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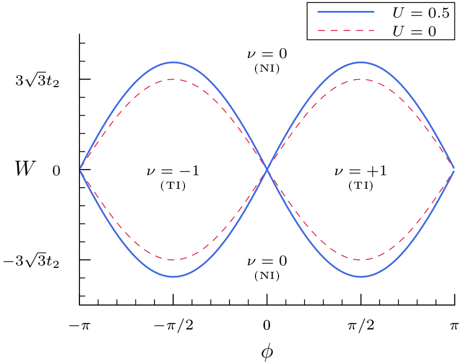
- For each $\omega = \pm$, unique solution to $m_{R,\omega} = 0$ (no bifurcation).

Renormalized transition curves



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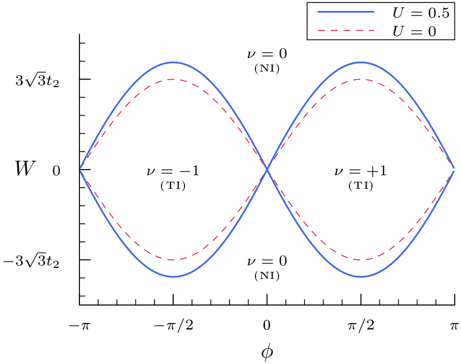

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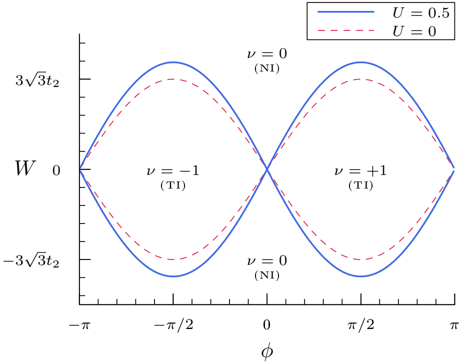

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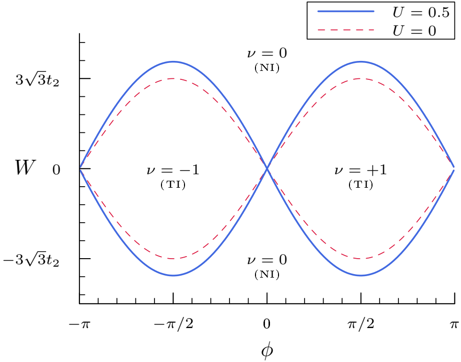

- Away from the blue curve the correlations decay exponentially fast. On the blue curve the decay is algebraic $\Rightarrow$ chiral semi-metal.
- Repulsive interactions enhance the topological insulator phase
- We rigorously exclude the appearance of novel phases in the vicinity of the unperturbed critical lines.

Outline

1. Overview
2. Introduction
3. The model and the main results
4. Sketch of the proof

Main strategy, I

Step 1: We employ constructive field theory methods (fermionic Renormalization Group: determinant expansion, Gram-Hadamard bounds, ...) to prove that:

- the Euclidean correlation functions are analytic in U, uniformly in the renormalized mass, and decay at least like $|\mathbf{x}|^{-2}$ at large space-(imaginary)time separations.

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The result builds upon the theory developed by Gawedski-Kupiainen, Battle-Brydges-Federbush, Lesniewski, Benfatto-Gallavotti, Benfatto-Mastropietro, Feldman-Magnen-Rivasseau-Trubowitz, ..., in the last 30 years or so.

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$$\hat{S}_2(k_0, \vec{p}_F^{\,\omega} + \vec{k}') =$$
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Step 2: Combining the existence of the g.s. euclidean correlations with a priori bounds on the correlation decay at complex times $t \in \mathbb{C}^+$, we infer the analyticity of correlations for $t \in \mathbb{C}^+$ (via Vitali's theorem)

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Next, using the $(\mathrm{Re}\ t)^{-2}$ decay in complex time, we perform a Wick rotation in the time integral entering the definition of $\sigma_{ij}(U)$: the integral along the imaginary time axis is the same as the one along the real line or, better, as the *limit of the integral along a path shadowing from above the real line*. Existence and exchangeability of the limit follows from Lieb-Robinson bounds.

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The general strategy is analogous to [Coleman-Hill '85]: "no corrections beyond 1-loop to the topological mass in QED_{2+1}."

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  - Effect of long-range interactions (e.g., static Coulomb)?

Thank you!